On the Scaling of Congestion in the Internet Graph*

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ABSTRACT

As the Internet grows in size, it becomes crucial to understand how the speeds of links in the network must improve in order to sustain the pressure of new end-nodes being added each day. Although the speeds of links in the core and at the edges improve roughly according to Moore's law, this improvement alone might not be enough. Indeed, the structure of the Internet graph and routing in the network might necessitate much faster improvements in the speeds of key links in the network.

In this paper, using a combination of analysis and extensive simulations, we show that the worst congestion in the Internet AS-level graph in fact scales poorly with the network size ($n^{1+\Omega(1)}$, where n is the number of nodes), when shortest-path routing is used to route traffic between ASes. We also show, somewhat surprisingly, that policy-based routing *does not* exacerbate the maximum congestion when compared to shortest-path routing.

Our results show that it is crucial to identify ways to alleviate this congestion to avoid some links from being perpetually congested. To this end, we show that the congestion scaling properties of Internet-like graphs can be improved dramatically by introducing moderate amounts of redundancy in the graph in terms of parallel edges between pairs of adjacent nodes.

Categories and Subject Descriptors

C.2 [**Computer Systems Organization**]: Computer-Communication Networks; C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design

General Terms

Congestion, Routing

Keywords

shortest path routing, power-law graphs, congestion

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1. INTRODUCTION

The Internet grows in size every day. As time progresses, more end-hosts are added to the edge of the network. Correspondingly, to accommodate these new end-hosts, ISPs add more routers and links. History has shown that the addition of these links maintains the power law properties of the Internet topology [9]. The addition of new end-hosts places a greater load on the network as a whole. Fortunately, the improvement of network technology operates over the same time period. We expect the network links at the edge and core of the network to improve by a similar performance factor as the growth of traffic over time, since they both typically follow similar Moore's Law-like technology trends.

Unfortunately, due to the topology of the network and behavior of Internet routing, the increase in load may be different on different links. As a result, it may be necessary for the speed of some hot-spot links in the network to improve much more quickly than others. If this is true, then these parts of the network are likely to eventually become bottlenecks and the network has poor scaling properties. In such a situation, we would either need to adjust the routing behavior, remove the power law nature of the topology or accept that end-to-end network performance will not improve as rapidly as individual links. If, on the other hand, the worst congestion scales well with the network size then we can expect the network to continue to operate as it does now.

In this paper, we perform a preliminary study of how the maximum congestion in the Internet scales with the network size, under reasonably realistic models of network evolution and inter-domain routing. Our study focuses on the Internet *AS-level graph* and employs a combination of simple analysis and extensive simulations. While the detailed simulation results establish the exact scaling properties, the analysis serves to provide preliminary intuition behind the observed scaling of congestion.

Our analysis is based on the Preferential Connectivity [7] model of network evolution and a simple model of traffic in which a unit amount of flow between every pair of nodes is routed along the shortest path between them. It provides a back-of-the-envelop estimate of the worst congestion in the AS-level graph. Our simulation observations are based both on real and on synthetically generated *AS-level* topologies and synthetic traffic matrices. Through our simulations, we also investigate the impact of several key factors on the scaling of congestion in the network, such as variants of the inter-domain routing algorithm, alternate skewed traffic matrices, and finally, alternate degree structures of the underlying topology.

Contributions of our work. The key contribution of our paper is to show that *the maximum congestion in Internet-like graphs scales poorly with the growing size of the graph*. Specifically, the maximum congestion for shortest path routing and uniform traffic matrices is at least as bad as $n^{1+\Omega(1)}$, with the exponent depending on the exponent of the power law degree distribution of the graph[1]. Our simulations show that policy routing in the AS graph results in roughly the same maximum congestion as shortest path routing, *but certainly not worse*. When alternate, non-uniform traffic models are considered, the congestion scaling properties of power law graphs worsen substantially. We also show that in terms of the maximum congestion, power law trees are considerably worse than power law graphs. In contrast, graphs with exponential degree distribution have very good congestion properties.

Another key contribution of our paper is the discussion of simple guidelines that result in a dramatic improvement in the congestion scaling properties of Internet-like graphs. We show that when parallel links are added between adjacent nodes in the network according to simple functions of their degrees (e.g., the minimum of the two degrees), the maximum congestion in the resulting graph scales linearly.

Limitations of our work. We would like to mention that our results may not hold in general for all power law graphs. Our results (both simulation-based and analytical) are meant for graphs representing Internet connectivity at the AS level. In particular, our analytical results hold for Internet-like graphs arising from the preferential connectivity model. These results, for example, may not apply to power-law *random graphs* [3].

As mentioned earlier, our analysis is approximate and makes several simplifying assumptions. A more complete and rigorous analysis of the scaling of congestion when shortest path routing is employed in preferential connectivity graphs is a challenging problem and is left as future work.

Note also that while the preferential connectivity model is known to yield graphs with a similar degree distribution as the AS-level graph, it is not clear whether the model accurately captures the AS-level connectivity dynamics (e.g., economic considerations for peering). That said, our simulations on measured AS-level graphs show that our key observations hold for the existing AS graph. Therefore, if the current dynamics of connectivity between ASes continues to hold in the future, we can expect our results to hold for future AS-level graphs too.

Analyzing the router-level graph is much harder compared to analyzing the AS-level graph due to three reasons: (1) Not much is known about the topology of Internet's router-level graph. Most existing maps of the Internet's router-level topology are considered incomplete; (2) IP-level routing cannot be modeled easily using shortest path routing or simple inter-domain policy-based routing, since this would require knowledge of traffic engineering employed by ASes in the Internet; (3) Finally, some researchers have used power-law graphs resulting from probabilistic models such as [3, 7] to approximate the router-level connectivity (see for example [22]). However, recent work has shown that such models are error-prone since they do not explicitly consider the technological and economic constraints or trade-offs behind router interconnections [15]. Graphs arising from such trade-offs are referred to as *Heuristically Optimal Topologies*. However, there are no analytically-tractable models for generating such topologies. Due to these constraints we leave a thorough analysis of the router-level interconnection as future work.

Paper organization. The rest of the paper is structured as follows. We discuss related work in Section 2. In Section 3, we formalize the problem and discuss our simulation set-up. Preliminary analytical intuition behind the scaling of congestion is presented in Section 4. Section 5 presents the results from our simulations. In Section 6, we discuss the implications of our results on network design. Finally, in Section 7, we conclude the paper.

2. RELATED WORK

In the past, there have been several research efforts aimed at studying the properties of large-scale, Internet-like graphs. Of these, one class of studies has proposed various models of graph evolution that result in a power law degree distribution. Notable examples include the power law random graph model of Aiello *et. al.* [3], the bicriteria optimization model of Fabrikant *et. al.* [8] and the Preferential Connectivity model of Barabasi and Albert [7, 5]. Another class of studies in this category [9, 20, 22] is aimed at analyzing the properties of power law graphs. However, most of these are based on inferences drawn from measurements of real data. The primary application of this latter class of studies is to construct realistic generators [16, 23, 22] for Internet-like graphs. Our theoretical analysis is based on the Preferential Connectivity model of Barabasi and Albert [7]. Our simulations use topologies generated synthetically using Inet-3.0 [23].

The problem of characterizing congestion in graphs, and specifically designing routing schemes that minimize congestion, has been studied widely in approximation and online algorithms. The worst congestion in a graph is inversely related to the maximum concurrent flow that can be achieved in the graph while obeying unit edge capacities. The lat-

[1] There is some disagreement about whether a power law correctly models the degree distribution of the Internet graph. However, it is widely agreed that the distribution is heavy-tailed. While our main results (specifically, simulation results) focus on power law distributions, we believe that they hold equally well for other such heavy-tailed distributions (e.g. Weibull).

ter is, in turn, related to a quantity called the cut ratio of the graph. Aumann *et. al.* [6] characterize the relationship between maximum concurrent flow and cut ratio[2] and Okamura *et. al.* [19] give bounds on the cut ratio for special graphs. Algorithmic approaches to the problem (see [13, 14] for a survey) use a multi-commodity flow relaxation of the problem to find a fractional routing with good congestion properties. Although fairly good approximation factors have been achieved for the problem, most of the proposed routing schemes are not distributed, involve a lot of book-keeping, or involve solving large linear programs, which makes them impractical from the point of view of routing on the Internet. Therefore, we choose the approach of analyzing the congestion achieved from using widely implemented routing schemes such as shortest path or BGP-policy based routing.

Perhaps the work that bears closest resemblance to ours is that of Gksanditis *et al.* [12]. Using arguments from max-flow min-cut theory, their paper shows that graphs obeying power law degree distribution have good expansion properties in that, they *allow* routing with $O(n \log^2 n)$ congestion, which is close to the optimal value of $O(n \log n)$ achieved by regular expanders. In addition, based on simulations run over Inet-generated topologies, the paper concludes that the congestion in power law graphs scales almost as $O(n \log^2 n)$, *even when shortest path routing is used*. The paper also shows that policy routing results in worse congestion. In a follow-up paper, Mihail *et al.* [17] prove similar results on the expansion properties of power law graphs generated using the Preferential Connectivity model.

Our work is different from both these papers in several key aspects, a few of which we identify below. First, the theoretical analysis in [12] and [17] does not restrict the routing to shortest path and, in fact, assumes an optimal routing algorithm that minimizes congestion. We give evidence to the fact that, when shortest path routing is employed, power law graphs exhibit poor congestion scaling properties. The maximum congestion scales as $n^{1+\Omega(1)}$. We confirm this via detailed simulation experiments. In addition, our simulations also show that policy routing *does not* worsen the maximum congestion in the network contrary to the conclusion in [12]. The evaluations of policy routing and shortest path routing in [12] only consider graphs with a small number of nodes, approximately 10,000 nodes for policy routing graphs (due to the dependence on real AS graphs) and only 23,000 for the shortest path routing graphs. Our simulations, on the other hand, consider graphs of up to 50000 nodes. Finally, we also consider the impact of different traffic workloads and deployments of parallel links on the scaling properties of the network.

3. METHODOLOGY

We use combinatorial/probabilistic arguments over a simple model of the network combined with extensive simulations to understand the congestion properties of Internet-like graphs. In what follows, we first give a precise formulation of the problem, laying out the key questions we seek to address via analysis. We also describe the set-up for the simulations we use to corroborate and extend our analytical arguments.

[2] The maximum concurrent flow that can be achieved in a graph is always within a factor of $O(\log n)$ of the cut ratio, where n is the number of nodes.

3.1 Problem Statement

Let $G = (V, E)$ be an unweighted graph, representing the Internet AS-level graph, with $|V| = n$. Let d_v denote the total degree of a vertex v in G. We are given three key aspects pertaining to the graph G: the degree distribution of the graph, the routing algorithm used by the nodes in the graph to communicate with each other and the traffic demand matrix determining the amount of traffic between pairs of nodes in the graphs. We give precise definitions of these three aspects, in turn, below.

In our paper we will mostly be concerned with graphs having a power law degree distribution, defined below.

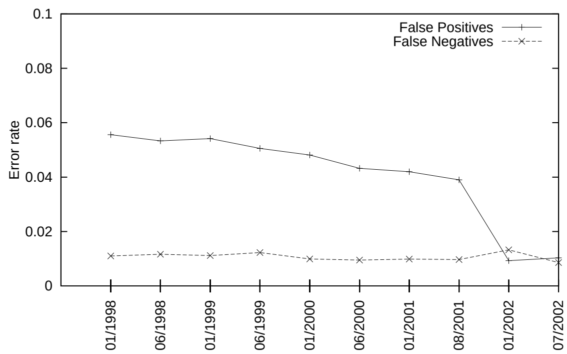
DEFINITION 1. *We say that an unweighted graph G has a power law degree distribution with exponent α, if for all integers d, the number of nodes v with $d_v \geq d$ is proportional to $d^{-\alpha}$.*

Similarly, graphs with exponential degree distribution are those in which the number of nodes v with $d_v \geq d$ is proportional to $e^{-\beta d}$, for all integers d. Henceforth, we will refer to such graphs as power law graphs and exponential graphs respectively.

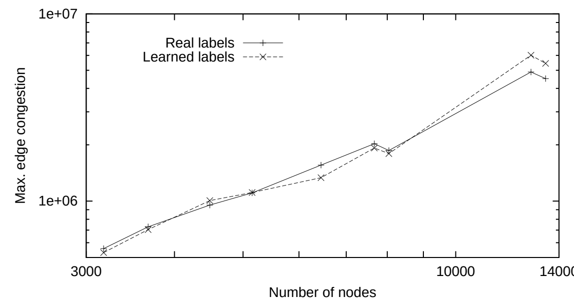
Let $\mathcal{S}$ denote a routing scheme on the graph with $\mathcal{S}_{u,v}$ representing the path for routing traffic between nodes u and v. We consider two different routing schemes in this paper:

1. **Shortest Path Routing:** In this scheme, the route between nodes u and v is given by the shortest path between the two nodes in the graph G. When there are multiple shortest paths, we consider the maximum degree of nodes along the paths and pick the one with the highest maximum degree. This tie-breaking rule is reflective of the typical policies employed in the Internet—higher degree nodes are typically much larger and much more well-provisioned providers than lower degree nodes and are in general used as the primary connection by stub networks. In Section 5.3, we consider alternate tie-breaking schemes such as random choice and favoring lower degree nodes, and show that the tie-breaking rule does not effect our results much.
2. **Policy Routing:** In this scheme, traffic between nodes u and v is routed according to BGP-policy. We classify edges as peering edges or customer-provider edges (that is, one of the ASes is a provider of the other). Typically, ASes in the Internet only provide transit for traffic destined to their customers, if any. This implies that no AS will carry traffic from its peer to another of its peers or to its provider. Similarly, no AS will carry traffic from one of its providers to one of its peers or to its provider. These rules together give rise to "valley-free" routing, in which each path contains a sequence of customer to provider edges, followed by at most one peering edge, followed by provider to customer edges. For a detailed description of the mechanism, the reader is referred to [21].

A traffic vector τ is a vector containing $\binom{n}{2}$ non-negative terms, with the term corresponding to (u, v) signifying the



(a) Accuracy of Stub Identification



(b) Accuracy of Edge Labeling

Figure 1: Accuracy of heuristics: The graph on the left shows the accuracy of our simple stub identification algorithm. The graph on the right shows the error in the maximum congestion due to our machine-learning based edge-classification algorithm.

amount of traffic between the nodes u and v. The congestion on an edge e due to traffic vector τ and routing scheme $\mathcal{S}$ is given by the sum of the total amount of traffic that uses the edge e: $\mathcal{C}_{\tau,\mathcal{S}}(e) = \sum_{(u,v):e\in\mathcal{S}_{u,v}} \tau(u,v)$.

We define the edge congestion due to traffic vector τ and routing scheme $\mathcal{S}$ to be the maximum congestion on any edge in the graph:

$$\text{EDGE-CONGESTION}_{\tau,\mathcal{S}}(G) = \max_{e\in E} \mathcal{C}_{\tau,\mathcal{S}}(e)$$

In this paper, we are interested in quantifying the congestion in a graph with power law degree distribution, for shortest path and policy routing schemes, due to various different traffic vectors. Specifically, we consider the following three traffic vectors:

1. **Any-2-any:** This corresponds to the all 1s traffic vector, with a unit traffic between every pair of nodes.
2. **Leaf-2-leaf:** In order to define this model, we classify nodes in the graph as *stubs* and *carriers*. Stubs are nodes that do not have any customers. In other words, consider directing all customer-provider edges in the graph from the customer to the provider. Peering edges are considered to be bidirected edges. Then, vertices with no incoming edges (corresponding to ASes with no customers) are called stubs or leaves in the graph. In this model, there is a unit of traffic between every pair of stubs in the graph.
3. **Clout:** This model is motivated by the fact that "well-placed" sources, that is, sources that have a high degree and are connected to high degree neighbors, are likely to send larger amounts of traffic than other sources. Accordingly, in this case, $\tau(u,v) = f(d_u, c_u)$, where u and v are stubs, c_u is the average degree of the neighbors of u and f is an increasing function. As in the previous case, there is no traffic between nodes that are not stubs. In this paper, we only use the function $\tau(u,v) = f(d_u, c_u) = d_u c_u$ for stubs u, v.

Admittedly, our choice of the models for Internet routing, as well those for Internet traffic matrices, are somewhat unrealistic. However, we still use them in our analysis for reasons of simplicity, analytical tractability and for lack of realistic Internet-wide traffic traces. We believe that these models can be enriched with support from appropriate wide-area measurements, and leave this as future work. Despite these drawbacks, our work is significant, in that, it is the first such effort to expose a fundamental weakness in the macroscopic design of the Internet.

3.2 Simulation Set-up

Our simulations serve two purposes: (1) to corroborate our theoretical results, and, (2) to characterize the congestion in more realistic network models than those considered in our analysis.

Our simulations are run on two different sets of graphs. The first set of graphs contains maps of the Internet AS topology collected at 6 month intervals between Nov. 1997 and April 2002, available at [2]. The number of nodes in any graph in this set is at most 13000, the maximum corresponding to the April 2002 set. The second set of graphs contains synthetic power law graphs generated by Inet-3.0 [23]. In this set, we generate graphs of sizes varying from $n = 4000$ to 50000 nodes. In all our simulations, for any metric of interest, for each n, we generate 5 different graphs of n nodes[3] and report the average of the metric on the 5 graphs.

As pointed out in Section 3.1, in order to implement the leaf-2-leaf and clout models of communication, we need to identify stubs in the network (note that these might have a degree greater than 1). Additionally, in order to implement policy routing, we need to classify edges as peering or non-peering edges. In order to do so, for the real AS graphs, we employ the relationship inference algorithms of Gao [11] to label the edges of the graphs as peering or customer-provider edges. These algorithms use global BGP tables [1] to infer relationships between nodes. Then, we use these relationships to identify stubs (as nodes that are not providers of any other node). Henceforth, we shall refer to the real AS

[3] By varying the random seed used by the Inet graph generator.

graphs as *accurately labeled real* graphs (ALRs). Labeling edges and identifying stubs in the synthetic graphs of Inet is more tricky since we do not have the corresponding BGP information. We will refer to synthetic graphs, labeled using the algorithms described below, as *heuristically labeled synthetic* graphs (HLSs). We use different algorithms for classifying nodes (this is key to implementing leaf-to-leaf communication) and edges (this is key to implementing policy routing in synthetic graphs). We discuss each in turn below.

Stub identification. Here is how we identify stubs in synthetic graphs: For any edge $e = (v_1, v_2)$, we assign v_1 to be the provider of v_2 whenever $degree(v_1) \geq degree(v_2)$. Notice that we do not explicitly identify peering edges (although edges between nodes of identical degree will be bidirectional). We then identify stubs in graphs labeled as above.

We test the accuracy of this stub-identification algorithm on real AS graphs by comparing the labels produced by our algorithm to the true labels of ALRs, and compute the fraction of false positives and false negatives[4] in these. The results (see Figure 1(a)) show that our simple algorithm has very low error rate. Notice that the inference algorithms of Gao [11] have some error intrinsically and hence some of the labels on the ALRs might actually be inaccurate.

Edge classification. Although for the purpose of classifying nodes, we simply consider all edges in the graph to be customer-provider edges, this simple scheme is not useful for the purposes of edge classification – it results in a significant error on the maximum congestion in real graphs employing policy routing. In order to improve the accuracy of labeling edges, we resort to machine learning algorithms.

Employing a good machine learning algorithm for the classification proves to be a tough task, because there is a huge bias towards customer-provider edges in the graphs (roughly 95% of the edges are customer-provider edges). We use the 3-Nearest Neighbor [18] algorithm for classifying edges as peering or non-peering: each edge in the unlabeled graph is classified as a peering edge if among the three edges most similar to it in the labeled graph, at least two are peering edges. Similarity between edges is judged based on the degrees of their respective end points and neighboring vertices. We measure the accuracy of the procedure by applying it to real graphs and then comparing the classification with true labels.

Our machine learning algorithm gives only 20% accuracy on peering edges and about 95% accuracy on customer-provider edges. However, for the purposes of computing the worst congestion in the graph, this low accuracy of labeling is in fact enough. Indeed, as shown in Figure 1(b), labeling real graphs using our algorithm results in an error of less than 10% in the worst congestion (while employing policy routing) in comparison with the congestion computed on ALRs.

[4]False positives are nodes that are identified as stubs by the algorithm, but are not stubs in the ALR. False negatives are stubs in the ALR that are not identifies as stubs by the algorithm.

More importantly, the growth in congestion is identical in the two cases.

We also report simulation results for congestion in power law trees and exponential topologies. A comparison of the former with power law graphs gives an insight into the significance of density of edges in the graph. The latter model is interesting because most generative models for power law topologies result in exponential distributions in the "fringe" cases. Our tree topologies evolve according to the Preferential Connectivity model [7]. To generate exponential degree distributions, we modify Inet-3.0 to generate an exponential degree distribution first and then add edges in Inet's usual way. For a given n, the exponent β for the exponential graphs on n nodes is chosen such that the total number of edges in the exponential graph is very close to that of the corresponding power law graph on n nodes[5]. Note that due to a lack of real data for exponential graphs, we do not have a good way of labeling edges and nodes in them. We do not perform experiments with policy routing or the leaf-2-leaf and clout traffic models for them.

4. ANALYSIS

In this section, we give theoretical evidence showing that the expected maximum edge congestion in a power law graph grows as $\Omega(n^{1+\frac{1}{\alpha}})$ with n, when we route a unit flow between all pairs of vertices over the shortest path between them.

We consider the Preferential Connectivity Generative Model of Barabasi and Albert [7]. For completeness, a brief description of the model is given below. In addition, we report results from experiments conducted to validate that the observations made below hold not just for the Preferential Connectivity Model, but also for Internet-like graphs generated by Inet-3.0.

The Preferential Connectivity model is as follows: We use a fixed constant parameter k. We start with a complete graph on $k+1$ nodes. We call this set of nodes the **core** of the graph. Let the graph at time i be denoted G^i. At time step $i+1$, one node is added to the network. This node picks k nodes at random and connects to them. Each vertex v has a probability $\frac{d_v^i}{D^i}$ of getting picked, where d_v^i is the degree of the vertex at time i, and D^i is the total degree of all nodes at time i.

At the end of n steps, with $k = 3$, this process is known to generate a power law degree distribution. We will use the fact that in a power law graph with exponent $\alpha > 1$, the maximum degree node has degree $n^{1/\alpha}$.

In order to obtain a bound on the congestion of a power law graph, our plan is roughly as follows. We consider the edge between the two highest degree nodes in the core—s_1 and s_2. Call this edge e^*. For every vertex v in the graph, we consider the shortest path tree T_v rooted at vertex v. We give evidence below that, in expectation, $\Omega(n)$ such trees contain the edge e^*. Moreover, we show that in these trees, the expected number of nodes in the subtree rooted at edge e^* is at least $\Omega(n^{\frac{1}{\alpha}})$.

[5]We employ heuristic hill-climbing to estimate the value of the exponent β that minimizes error in the number of edges.

This gives us a bound on the congestion in the following way: the routes taken by each connection are precisely those defined by the above shortest path trees; thus the congestion on any edge is the sum of congestions on the edge in these shortest path trees. Now, as described above, in $\Omega(n)$ shortest path trees, the congestion on edge e^* is expected to be at least $\Omega(n^{\frac{1}{\alpha}})$. Therefore, the total congestion on edge e^* is at least $\Omega(n^{1+\frac{1}{\alpha}})$. Note that e^* is not necessarily the most congested edge in the graph, so the maximum congestion could be even worse than $\Omega(n^{1+\frac{1}{\alpha}})$. We get the following hypothesis:

Hypothesis 1. *The expected value of the maximum edge congestion in a power law graph with exponent α grows as $\Omega(n^{1+\frac{1}{\alpha}})$ with n, when we route a unit flow between all pairs of vertices over the shortest path between them.*

In the following, the distance between two nodes refers to the number of hops in the shortest path between the two nodes. We make a few technical assumptions. We assume that $1 < \alpha < 2$, and s_1 and s_2 are the highest degree nodes in the graph. For reasonably "small" numbers h, we assume that for any node v in the graph, the number of nodes within distance h of v is less than the number of nodes within distance h of s_1. In other words, s_1 is centrally placed in the graph. Here, "small" refers to distance around s_1 that contains lesser than half the nodes. These assumptions are justified by experimental evidence and some prior analysis [10] of the preferential connectivity generative model.

We begin with a technical claim.

Claim 1. *Let r be the maximum integer for which at least $\frac{n}{2}$ vertices lie at a distance $r+1$ or beyond from s_1. Then, $\Omega(n)$ nodes lie within distance $r-1$ of every node in the core of the graph.*

Proof. We prove that at least $\Omega(n)$ nodes lie within a distance $r-2$ of s_1. Then, since all vertices in the core are neighbors of s_1, these $\Omega(n)$ nodes lie within a distance $r-1$ of any vertex in the core of the graph. We begin by showing that at least $\Omega(n)$ nodes lie within a distance r of s_1, and then extend this to nodes at distance $r-1$ and $r-2$. Let level i denote the set of nodes at distance exactly i from s_1.

Remove from the graph all vertices that are at level $r+2$ or higher. The remaining graph has at least $\frac{n}{2}$ vertices, by the definition of r. Now, assume that there are at least $\frac{n}{10}$ vertices at level $r+1$, otherwise, we already have $> \frac{2n}{5}$ nodes in levels 0 through r, implying that $\Omega(n)$ nodes lie within distance r of s_1.

Now, let the number of nodes at level r be x. All the nodes in level $r+1$ in the residual graph are connected to nodes in level r. So, their number is at most the size of the neighbor set of level r. Now, in the best possible case, the nodes in level r could be the highest degree nodes in the graph. In this case, the minimum degree of any node in level r is given by y with $ny^{-\alpha} = x$. We get $y = \left(\frac{n}{x}\right)^{\frac{1}{\alpha}}$.

Now, the size of the neighborhood of level r is at most the total degree of nodes in the level. This is given by

$$\begin{aligned}\int_y^{n^{\frac{1}{\alpha}}} z\alpha n z^{-\alpha-1} dz &= \frac{\alpha n}{\alpha-1}\left(y^{1-\alpha} - n^{\frac{1}{\alpha}-1}\right) \\ &= \frac{\alpha n^{\frac{1}{\alpha}}}{\alpha-1}(x^{1-\frac{1}{\alpha}} - 1)\end{aligned}$$

This quantity is at least $\frac{n}{10}$ by our assumption above. Thus we get that $x = \beta n$, where $\beta = \left(\frac{1}{10}(1-\frac{1}{\alpha})\right)^{\frac{\alpha}{\alpha-1}}$. This is a constant fraction of n.

Now, we can apply the same technique to compute the number of nodes at level $r-1$ and then, $r-2$. We get that the number of nodes at level $r-2$ is at least $\left(\beta^{\alpha}(1-\frac{1}{\alpha})\right)^{\frac{\alpha}{(\alpha-1)^2}} n$, with β as given above. □

Let r be the distance defined by the above lemma. Let V_r denote the set of nodes that are within distance $r-1$ of every node in the core of the graph (see Figure 2). By Claim 1, we have $|V_r| = \Omega(n)$. Now, the rest of our argument has two parts. First we consider shortest path trees T_v corresponding to $v \in V_r$, that contain the edge e^*. We show that in each such tree, the congestion of e^* is high, viz., $\Omega(n^{1/\alpha})$.

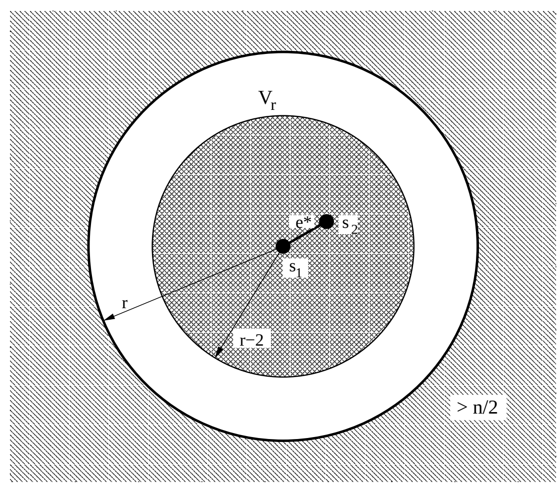


Figure 2: A pictorial view of the graph and the set V_r

Claim 2. *Let T_v be a shortest path tree, corresponding to $v \in V_r$, that contains the edge e^*. Then the expected congestion on edge e^* in this tree is $\Omega(n^{1/\alpha})$.*

Proof. Without loss of generality, let s_1 be closer to v than s_2. We show that the degree of s_2 in T_v is $\Omega(n^{1/\alpha})$. This implies the result. Let level i denote the set of nodes at distance i from v in the tree.

Let d be the distance between v and s_2. All neighbors of s_2 lie in levels $\geq d-1$ in the tree. Note that $d \leq r-1$.

Therefore by our assumption, the number of nodes lying at levels $\geq d+1$ in the tree is at least the number of nodes at distance r or greater from s_1. This number is at least $\frac{n}{2}$, by the definition of r. Let W denote the set of nodes that lie at levels $\geq d-1$ in the tree, and that arrived in the graph after step $\frac{n}{4}$. Note that there are at least $\frac{n}{4}$ nodes at level $d+1$ or higher that are in set W. Therefore, a constant fraction of the nodes in W lie at levels $\geq d+1$ in the tree.

First observe that at time step t, the number of neighbors of s_2 is roughly $t^{\frac{1}{\alpha}}$. When $t = \frac{n}{4}$, this number is $(\frac{n}{4})^{\frac{1}{\alpha}} < 0.5n^{\frac{1}{\alpha}}$, using $\alpha < 2$. This is half the total degree of s_2. So by removing the first quarter of the nodes entering the graph from consideration, we conclude that the number of neighbors of s_2 that arrive after step $n/4$ is at least half the total degree of s_2.

Now all neighbors of s_2 lie at levels $\geq d-1$ in the tree. Then, by the observation in the previous paragraph, we have that at least half of the neighbors of s_2 lie in the set W. Note that when a node in W entered the graph, the size of the graph varied between $\frac{n}{4}$ and n nodes. The probability that this node attached to s_2 varied between $n^{\frac{1}{\alpha}-1}$ and $(\frac{n}{4})^{\frac{1}{\alpha}-1} < 4n^{\frac{1}{\alpha}-1}$. Thus each node in W is roughly equally likely to attach to s_2 (within a factor of 4).

Now the degree of s_2 in the tree is at least the number of its neighbors in W that lie at levels $\geq d+1$. Using the fact that a constant fraction of the nodes in W lie at levels $\geq d+1$ in the tree and these are all roughly equally likely to connect to s_2, we get that a constant fraction of the neighbors of s_2 lie at levels $\geq d+1$ in the tree, in expectation. The result follows from the fact that the degree of s_2 is $\Omega(n^{\frac{1}{\alpha}})$. □

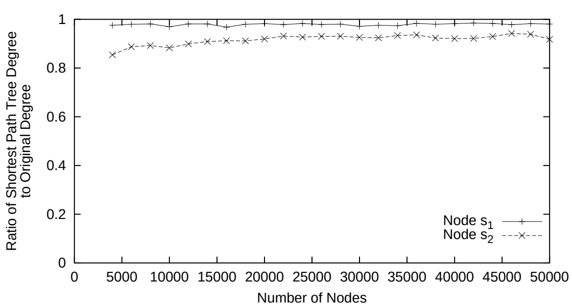


Figure 3: Ratio of degrees of s_1 and s_2 in a random shortest path tree to their degrees in the graph.

Figure 3 compares the degrees of the two highest degree nodes in the graph to their corresponding degrees in the shortest path tree corresponding to some random node v, in Inet-3.0 generated graphs.

Unfortunately, the graphs generated by Inet-3.0, have different values of α for different n. This is consistent with the observed behavior of the Internet, that α decreases with time. (We discuss this in further detail in the following section). In order to validate our theoretical claims and observe the asymptotic behavior of congestion for a fixed value of α, we modify the Inet-3.0 code, for the purposes of this section, so that it always uses a fixed value of $\alpha = 1.23$, instead of recalculating it for every value of n. Each reported value is an average over multiple runs of the simulation, corresponding to different random seeds used for generating the graphs.

We find that the ratio of the two degrees for s_1 is consistently above 0.9. Similarly, the ratio of the two degrees for s_2 is always above 0.8 and increasing. This is consistent with the findings of Claim 2.

Next we claim that a large number of trees T_v corresponding to $v \in V_r$ contain the edge e^*.

Consider the tree T_v for some node $v \in V_r$. This is essentially a breadth first tree starting from v. If s_1 and s_2 are at the same level in the tree, then the edge e^* is not contained in the tree. On the other hand, if the nodes are at different depths in this tree, let s_1 be closer to v without loss of generality. In this case, one shortest path from v to s_2 is via s_1 and since we break ties in favor of paths with high degree nodes, T_v will contain this path via s_1. This implies that e^* is contained in the tree. Thus, trees containing e^* correspond to those v that are not equidistant from s_1 and s_2. We now observe that there are $\Omega(n)$ nodes $v \in V_r$ that are not equidistant from s_1 and s_2, implying that $\Omega(n)$ trees T_v contain e^*.

Note that among the immediate neighbors of s_1 and s_2, the probability that these nodes are equidistant from s_1 and s_2 is exactly the probability that they connected to both s_1 and s_2 when they arrived. For a node arriving at time t, this is at most $O(t^{2(\frac{1}{\alpha}-1)})$. The expected number of nodes that connected to both s_1 and s_2 is therefore at most $O(\sum_t t^{2(\frac{1}{\alpha}-1)}) = o(n^{\frac{1}{\alpha}-1})$, and is therefore much less than the degree of either s_1 or s_2. If we could make a similar argument for every node in V_r, we would be done. The complication arises from noting that a node could become equidistant from s_1 and s_2 a long time after it has arrived, through a new node arriving later in the graph.

If we pick a random node in the graph, then conditioned on the fact that this node lies at a distance $d-1$, d or $d+1$ from s_2, there is at most a constant probability that this node lies at distance d from s_2. This is because using an argument congruent to that in Claim 1, we can show that the number of nodes at distance $d-1$ from s_2 is a constant fraction of the number of nodes at distance d.

Now, consider the nodes at distance l from s_1, where $l \leq r-2$. These are at least $\Omega(n)$ in number (Claim 1) and lie in V_r. Given that a node v is picked from this set, and is at distance l from s_1, v is at distance $l-1$, l or $l+1$ from s_2. Now using the argument above while conditioning on v being at distance l from s_1, we claim that the probability that this node lies at distance l from s_2 is at most a constant. This implies that $\Omega(n)$ nodes in this set are *not* equidistant from s_1 and s_2.

Figure 4 presents evidence to corroborate our claim. The figure plots the fraction of nodes that are equidistant from s_1 and s_2. Note that this fraction always lies below 0.4. Thus we can make the following observation:

OBSERVATION 1. *The expected number of shortest path trees T_v, corresponding to nodes $v \in V_r$, that contain the edge e^* is $\Omega(n)$.*

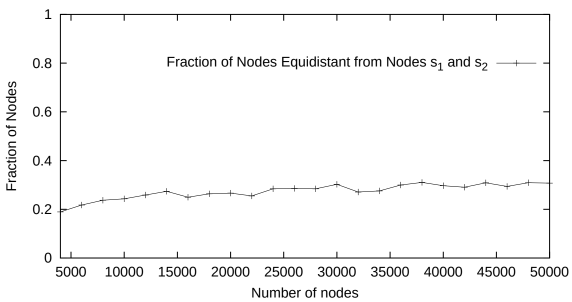


Figure 4: Fraction of shortest path trees that do not contain the edge e^*.

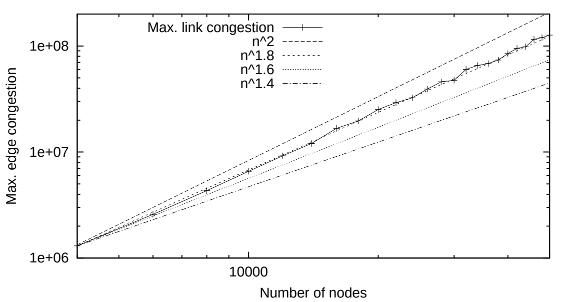


Figure 5: Maximum congestion as a function of n, in Inet-3.0 generated graphs, with $\alpha = 1.23$. The figure also plots four other functions to aid comparison – $n^{1.4}$, $n^{1.6}$, $n^{1.8}$, n^2.

This completes our analysis of the maximum congestion in a power law graph arising from preferential connectivity. As experimental evidence of our hypothesis, we plot the maximum congestion in graphs generated by Inet-3.0, as a function of the number of nodes in the graph, in Figure 5. Note that the maximum congestion scales roughly as $n^{1.8}$, which is exactly $n^{1+1/\alpha}$ for the given value of α. This corroborates our Hypothesis 1.

5. SIMULATION RESULTS

In this section, we present the results from our simulation study over Inet-generated graphs. Henceforth, we shall use the graphs generated by Inet 3.0 *as is*, that is, we do not alter the way Inet chooses α to depend on n. (Recall that, in contrast, the simulation results in the previous section use the modified Inet 3.0 code which employs the same value of α for all n. We *do not* show results for such graphs.) In what follows, we first show results for shortest-path routing, followed by policy-based routing. In both cases, we first present results for the any-2-any communication model, then for the leaf-2-leaf model and finally for the clout model.

5.1 Shortest-Path Routing

Figure 6(a) shows the maximum congestion in power law graphs generated by Inet-3.0 as a function of the number of nodes. We use the any-2-any model of communication here. From the trend in the graph, it is clear that *the maximum congestion in Internet-like power-law graphs scales as $n^{1+\Omega(1)}$ or worse*[6]. Notice also that the slope of the maximum congestion curve is slightly increasing. This may be explained as follows. As mentioned earlier, Inet-3.0 chooses the exponent of the power law degree distribution as a function of the number of nodes n: $\alpha = at + b$, where $t = \frac{1}{s} log \frac{n}{n_0}$, $a = -0.00324$, $b = 1.223$, $s = 0.0281$ and $n_0 = 3037$.[7] Notice that the absolute value of α decreases as n increases, and so, as our bound of $\Omega(n^{1+1/\alpha})$ suggests, the *slope* of the function on a log-log plot should steadily increase. In fact around $n = 28000$, α becomes less than 1 and at this point we expect the curve to scale roughly as n^2, which is the worst possible rate of growth of congestion.

The figure also shows the maximum congestion in power law trees and exponential graphs. The power law trees we generate, have the exponent α between 1.66 and 1.8, the value increasing with the number of nodes in the tree. These exponents are significantly higher than those of the corresponding power law graphs. Notice that the edge congestion on power law trees grows much faster as compared to graphs which is expected since trees have much fewer edges. Our analytical bound on the maximum congestion, which also holds for trees satisfying power law degree distributions, predicts the slope of the curve for trees to be at least 1.5, which is consistent with the above graph.

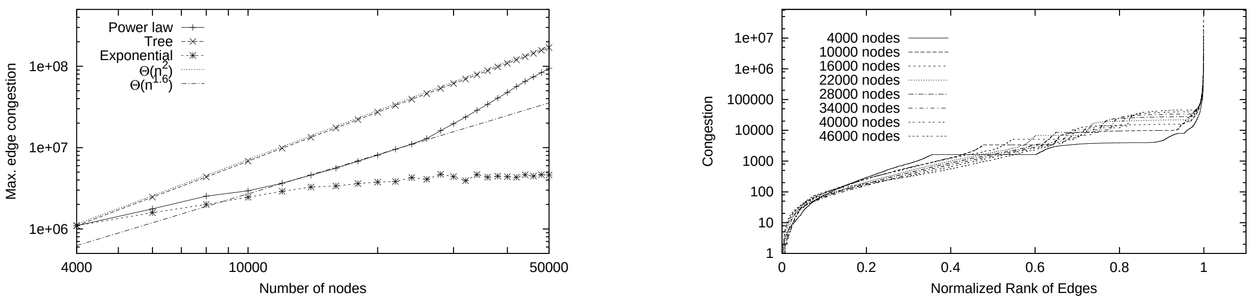
On the other hand, we notice that edge congestion in exponential graphs is much smaller compared to power law graphs. In fact, edge congestion in exponential graphs has a less than linear growth (i.e., scales as $O(n)$). This could be explained intuitively as follows: Recall that for each n, we choose the exponent β of the exponential distribution so as to match the total number of edges of the corresponding n-node power law graph. Because the power law distribution has a heavier tail compared to the exponential distribution, the latter has more edges incident on low degree nodes. Consequently, low degree vertices in an exponential graph are better connected to other low degree vertices. Edges incident on low degree nodes "absorb" a large amount of congestion leading to lower congestion on edges incident on high degree nodes. As n increases the degree distribution becomes more and more even, resulting in a very slow increase in congestion.

In Figure 6(b), we show the congestion across all links in a power law graph for varying numbers of nodes. Notice that at higher numbers of nodes, the distribution of congestion becomes more and more uneven.

The corresponding set of graphs for the leaf-2-leaf communication model is shown in Figure 7. The worst congestion is consistently about 0.8 times the worst congestion for the

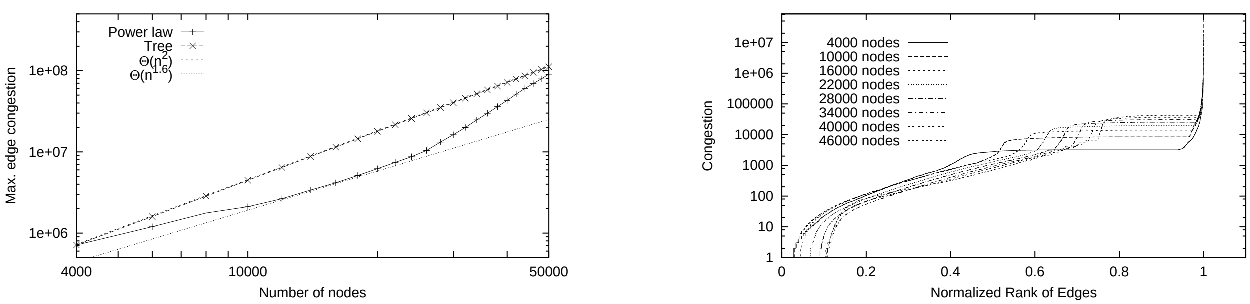
[6]Here, and henceforth, we use the term "Internet-like power-law graphs" or "Internet-like graphs" to refer to synthetically generated AS-level topologies.

[7]a, b and s are empirically determined constants. n_0 is the number of ASes in the Internet in November 1997.



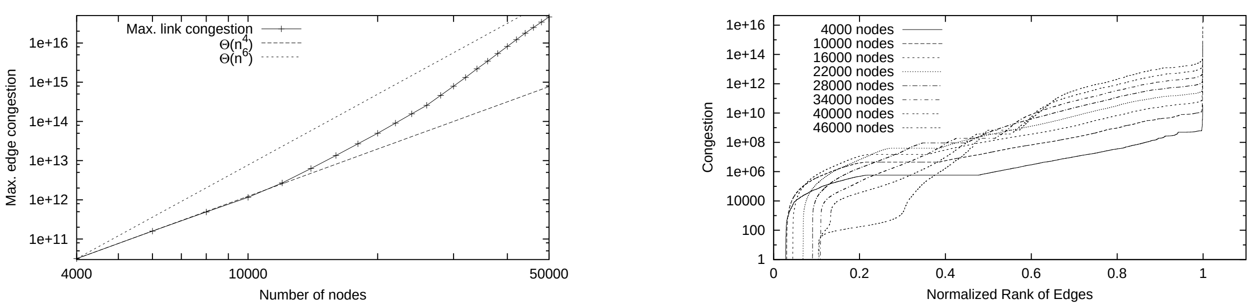
(a) Maximum edge congestion (b) Distribution of edge congestion

Figure 6: Edge congestion with shortest path routing and any-2-any communication: The figure on the left shows the maximum edge congestion. The figure on the right shows the distribution of congestion over all links, with the number of links normalized to 1 in each case. The figure on the left also plots the worst congestion for exponential graphs and preferential connectivity trees.



(a) Maximum edge congestion (b) Distribution of edge congestion

Figure 7: Edge congestion with shortest path routing and leaf-2-leaf communication



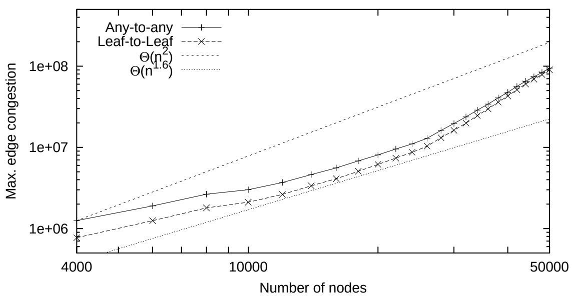
(a) Maximum edge congestion (b) Distribution of edge congestion

Figure 8: Edge congestion with shortest path routing and clout model of communication

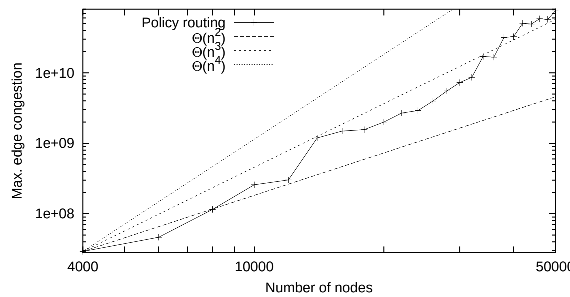
any-2-any model (not explicitly shown in the graph). The congestion across all the edges, plotted in Figure 7(b), also displays a similar trend as for the any-2-any model – the distribution becomes more uneven as the number of nodes increases.

The results for the clout model are more interesting with the resulting maximum congestion in the graph scaling much worse than before. Indeed, as Figure 8(a) shows, the maximum congestion scales worse than n^5. This is because the total traffic in the graph also grows roughly as $O(n^4)$. Again, as with the any-2-any model, the smaller absolute values of α in the graphs generated by Inet-3.0 for larger values of n is a plausible explanation for the increasing slope of the curve.

The graph of the congestion across all edges in this model, shown in Figure 8(b), is equally interesting. Compared to Figure 7(b) of the leaf-2-leaf model, Figure 8(b) looks very different. Visual inspection of the two figures reveals that the unevenness in congestion is much more pronounced in the clout model of communication. To summarize, *the non-*

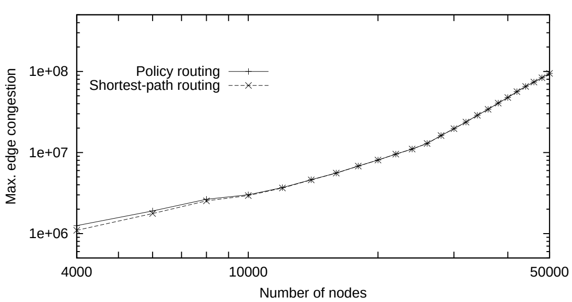


(a) Any-2-any and Leaf-2-leaf communication

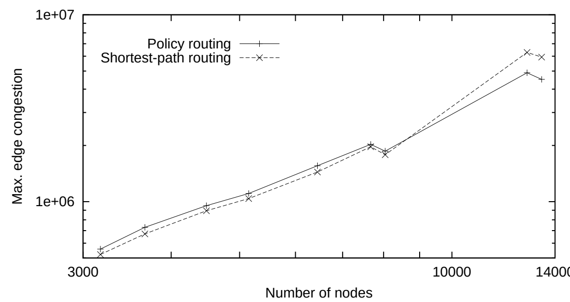


(b) Clout model

Figure 9: Maximum Edge congestion with policy-based routing in HLSs



(a) Edge congestion on synthetic graphs



(b) Edge congestion on real graphs

Figure 10: Comparison of edge congestion for shortest path and policy based routing in the any-2-any model

uniform traffic demand distribution only seems to exacerbate the already poor congestion scaling of the Internet-like graphs.

5.2 Policy-Based Routing

Figure 9 shows the maximum edge congestion for the three communication models when policy based routing is used. For the any-2-any and leaf-2-leaf models, shown in Figure 9(a), the maximum edge congestion scales almost identically to that for shortest path routing (compared with Figure 6(a) and 7(a)). However, somewhat surprisingly, for the clout model, congestion under policy based routing scales only as n^3 compared to over n^5 for shortest-path routing.

Figure 10(a) compares maximum congestion obtained for policy routing to that for shortest path routing. Notice that the two curves are almost overlapping, although policy routing seems to be slightly worse when the graph is small and gets better as the graph grows larger. This observation can be explained as follows: policy routing disallows certain paths from being used and could thus, in general, force connections to be routed over longer paths. This would increase the overall traffic in the network leading to higher congestion, especially for smaller numbers of nodes. However, as the size of the graph grows, there are more and more shortest paths available. As a result, the constraints placed by policy-based routing might not have any significant impact on the path lengths in the graph. In fact, at higher numbers of nodes, policy routing could provide better congestion properties, albeit only marginally different, than shortest path routing. This is because while shortest path routing always picks paths that go over high degree nodes, a fraction of these paths might not be allowed by policy routing as they could involve more than one peering edge. In this case, policy routing moves traffic away from the hot-spots, thereby, partially alleviating the problem.

In order to verify that the above observation is not just an artifact of our machine learning-based labeling algorithms, we plot the same curves for ALRs in Figure 10(b). These display exactly the same trend—policy routing starts out being worse than shortest path, but gets marginally better as n increases. To summarize, *policy routing does not worsen the congestion in Internet-like graphs, contrary to what common intuition might suggest. In fact, policy routing might perform marginally better than shortest path routing.*

5.3 Shortest Path Routing Variations

As mentioned in Section 3.1, in the shortest path routing scheme, whenever there are multiple shortest paths between two nodes, we pick the path that contains higher degree nodes to route the flow between them. It may appear that the poor congestion properties of powerlaw graphs are a result of this tie breaking rule, and an alternate rule that favors low degree nodes may perform better by alleviating the congestion on high degree nodes.

In order to confirm that our results are robust with respect to the tie-breaking rule, we performed the experiments with two variants of the tie-breaking rule: favoring paths that

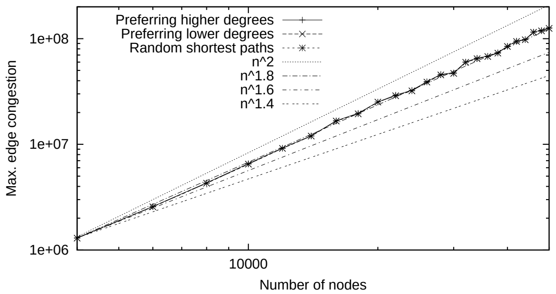


Figure 11: Edge congestion with shortest path routing and any-2-any communication, with $\alpha = 1.23$. The figure plots the three different variations of breaking ties in shortest path routing.

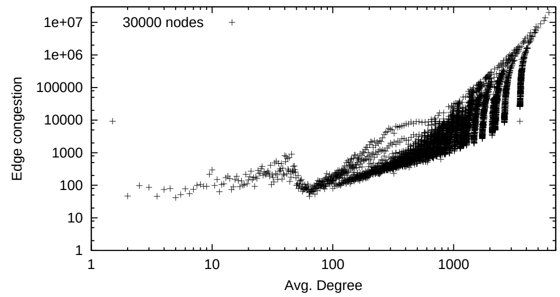


Figure 12: Edge Congestion versus the average degree of the nodes incident on the edge (any-2-any model with shortest path routing). The congestion is higher on edges with a high average degree.

contain lower degree nodes, and choosing a random shortest path when there is a choice of more than one. We report the results below.

For these experiments, we set α to be a constant value of 1.23 in Inet 3.0 and compare the resulting relations between maximum edge congestion and the number of nodes. As Figure 11 depicts, there is no noticeable difference between the three types of tie-breaking methods. The same holds true for Leaf-2-leaf and Clout models of traffic (results are omitted for brevity). This is because very few vertex pairs have multiple shortest paths between them. We thus conclude that our scheme of breaking ties by favoring paths containing higher degree nodes does not skew our results.

6. ALLEVIATING CONGESTION BY ADDING PARALLEL LINKS

Our theoretical evidence and simulation results have shown that the maximum congestion in Internet-like power-law graphs scales rather poorly in the graph size – $\Omega(n^{1+\Omega(1)})$. It is therefore possible that as the Internet AS-level graph grows in its size, the uniform scaling in the capacities of all links in the graph according to Moore's Law, might not be enough to sustain the increasing congestion in the graph. Our results show that edges between high degree nodes, which are typically peer edges between backbone carriers in the Internet core, are likely to get congested more quickly over time than the edges. In such a situation, to enhance the scaling properties of the network, it might become necessary to either change the routing algorithm employed by the nodes or alter the macroscopic structure of the graph.

We address the latter issue in this section. In particular, we examine ways in which additional links can be placed in the network, so as to contain the effect of bad scaling of the maximum congestion. Specifically, we consider the model in which each link can be replaced by multiple links (between the same pair of nodes) that can share the traffic load[8]. Ideally, we would like to provide sufficient parallel links between a pair of nodes, so that the total congestion on the corresponding edge divided equally among these parallel links, even in the worst case, grows at about the same rate as the size of the network. The number of parallel links between a pair of nodes may need to change as the network grows to achieve this goal. Notice that this change does alter the degree-structure of the graph, but the alteration is only due to increased connectivity between *already* adjacent nodes[9]. This does not require new edges between nodes that were not adjacent before.

In some ways, the network, at an AS level, already incorporates this concept of parallel links. For example, the power law structure of the AS graph only considers the adjacency of ASes: the link between Sprint and AT&T, for instance, is modelled by a single edge. However, in the real world the Sprint and AT&T ASes are connected to each other in a large number of places around the world. However, not much is known about the degree of such connectivity in the Internet today.

In order to guide the addition of parallel edges between adjacent nodes, we first observe that there is clear correlation between the average degree and edge congestion. Figure 12 plots the congestion of each edge against the average degree of the nodes on which it is incident, for shortest path routing on an Inet generated graph of 30000 nodes. The form of communication used here is any-2-any. The figure shows that edges incident on high degree nodes have much higher congestion than those incident on lower degree nodes. This suggests that *a good choice for the number of parallel links substituting any edge in the graph, could depend on the degrees of nodes which an edge connects.*

We examine several ways of adding parallel links based on the above observation. In particular, we let the number of links between two nodes be some function of the degrees of the two nodes and we consider the following functions: (1) sum of degrees of the two nodes, (2) product of the degrees of the two nodes, (3) maximum of the two degrees and, (4) minimum of the two degrees. For each of these functions, we compute the maximum relative congestion, that is, the maximum over all edges, of the congestion on the edge divided by the number of parallel links corresponding to each

[8]For results on alternate methods of alleviating congestion, please refer to a full version of this paper [4].

[9]Note that the routing is still done based on the original degrees of nodes.

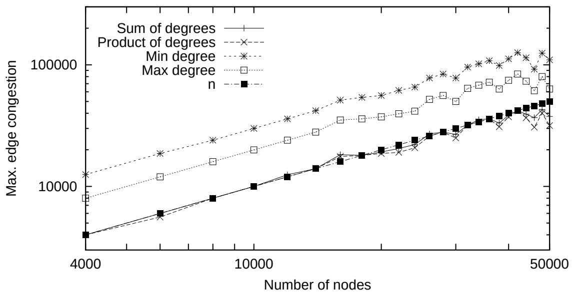


Figure 13: Maximum relative congestion for shortest path routing, any-2-any model, when parallel links are added to the graph using the sum, product and max functions.

edge. In what follows, we show simulation results about how the maximum relative congestion scales for shortest path routing on power law graphs within the any-2-any model of communication.

The results are shown in Figure 13. Notice that, surprisingly, when parallel links are added according to *any* of the above four functions *the maximum relative congestion in the graph scales linearly*. This implies that adding parallelism in the edges of Internet-like graphs according to the above simple functions is enough to ensure that the scaling of link capacities according to Moore's law can maintain uniform levels of congestion in the network and avoid any potential hot-spots.

7. SUMMARY

In this paper, we addressed the question of how the worst congestion in Internet-like graphs (specifically at the AS-level) scales with the graph size. Using a combination of analytical arguments and simulation experiments, we show that maximum congestion scales poorly in Internet-like power law graphs. Our simulation results show that the non-uniform demand distribution between nodes only exacerbates the congestion scaling. However, we find, surprisingly, that policy routing between adjacent ASes may not worsen the congestion scaling on power law graphs and might, in fact, be marginally better when compared to shortest-path routing.

Our results show that, with the current trend of the growth of the Internet, it is possible that some locations in the network might eventually become perpetual hot-spots. Fortunately, however, there is an intuitively simple fix to this problem. Adding parallel links between adjacent nodes (ASes) in the graph according to simple functions of their degrees will help the maximum congestion in the graph scale linearly. In this case, it might not be necessary for some links in the graph to grow in capacity at a faster rate than the others.

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