15-462 Computer Graphics I

Lecture 18

Radiosity

Measures of Illumination
The Radiosity Equation
Form Factors
Radiosity Algorithms
[Angel, Ch 13.4-13.5]

April 8, 2003
Frank Pfenning
Carnegie Mellon University

http://www.cs.cmu.edu/~fp/courses/graphics/

Local vs. Global Illumination

- Local illumination: Phong model (OpenGL)
  - Light to surface to viewer
  - No shadows, interreflections
  - Fast enough for interactive graphics
- Global illumination: Ray tracing
  - Multiple specular reflections and transmissions
  - Only one step of diffuse reflection
- Global illumination: Radiosity
  - All diffuse interreflections; shadows
  - Advanced: combine with specular reflection

Image vs. Object Space

- Image space: Ray tracing
  - Trace backwards from viewer
  - View-dependent calculation
  - Result: rasterized image (pixel by pixel)
- Object space: Radiosity
  - Assume only diffuse-diffuse interactions
  - View-independent calculation
  - Result: 3D model, color for each surface patch
  - Can render with OpenGL

Classical Radiosity Method

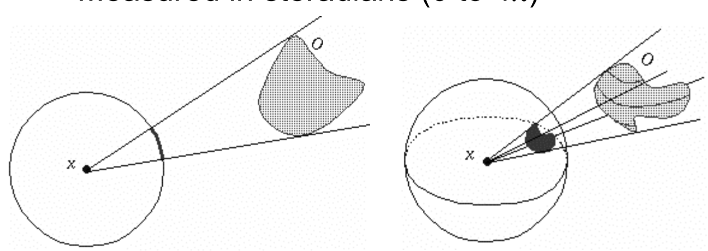
- Divide surfaces into patches (elements)
- Model light transfer between patches as system of linear equations
- Important assumptions:
  - Reflection and emission are diffuse
    - Recall: diffuse reflection is equal in all directions
    - So radiance is independent of direction
  - No participating media (no fog)
  - No transmission (only opaque surfaces)
  - Radiosity is constant across each element
  - Solve for R, G, B separately

Outline

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

Solid Angle

- 2D angle subtended by object O from point x:
  - Length of projection onto unit circle at x
  - Measured in radians (0 to 2π)
- 3D solid angle subtended by O from point x:
  - Area of of projection onto unit sphere at x
  - Measured in steradians (0 to 4π)



J. Stewart

Radiant Power and Radiosity

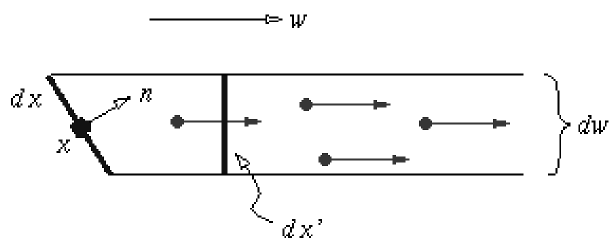
- Radiant power P
  - Rate at which light energy is transmitted
  - Dimension: power = energy / time
- Flux density Φ
  - Radiant power per unit area of the surface
  - Dimension: power / area
- Irradiance E: incident flux density of surface
- Radiosity B: exitant flux density of surface
  - Dimension: power / area
- Flux density at a point $\Phi(x) = dP/dx$

Power at Point in a Direction

- Radiant intensity I
  - Power radiated per unit solid angle by point source
  - Dimension: power / solid angle
- Radiant intensity in direction ω
  - $I(\omega) = dP/d\omega$
- Radiance $L(x, \omega)$
  - Flux density at point x in direction ω
  - Dimension: power / (area $\times$ solid angle)

Radiance

- Measured across surface in direction ω



J. Stewart '98

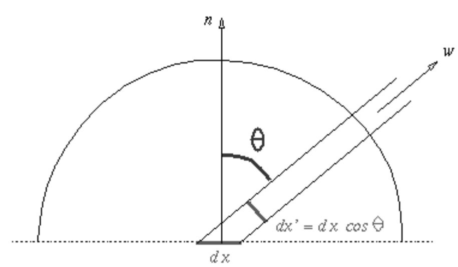
- For angle θ between ω and normal **n**

$$L(x,\omega) = \frac{d^2P}{d\omega\, dx'} = \frac{d^2P}{d\omega\ \cos\theta\, dx}$$

Radiosity and Radiance

- Radiosity B(x) = dP / dx
- Radiance $L(x,\omega) = d^2P / d\omega \cos\theta$ dx
- Let Ω be set of all directions above x

$$B(x) = \int_\Omega L(x,\omega)\ \cos\theta\, d\omega$$



Outline

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

Balance of Energy

- Lambertian surfaces (ideal diffuse reflector)
- Divided into n elements
- Variables
  - A_i Area of element i (computable)
  - B_i Radiosity of element i (unknown)
  - E_i Radiant emitted flux density of element i (given)
  - ρ_i Reflectance of element i (given)
  - $F_{j\,i}$ Form factor from j to i (computable)

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^{n} F_{ji} A_j B_j$$

Form Factors

- Form factor F_{ij}: Fraction of light leaving element i arriving at element j
- Depends on
  - Shape of patches i and j
  - Relative orientation of both patches
  - Distance between patches
  - Occlusion by other patches

Form Factor Equation

- Polar angles θ and θ' between normals and ray between x and y
- Visibility function v(x,y) = 0 if ray from x to y is occluded, v(x,y) = 1 otherwise
- Distance r between x and y

$$A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos\theta \ \cos\theta'}{\pi r^2} v(x,y)\, dy\, dx$$

Reciprocity

- Symmetry of form factor

$$A_i\, F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos\theta\ \cos\theta'}{\pi r^2} v(x,y)\, dy\, dx = A_j\, F_{ji}$$

- Divide earlier radiosity equation

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^{n} F_{ji} A_j B_j$$

by A_i

$$\begin{aligned} B_i &= E_i + \rho_i \sum_j (F_{ji} A_j / A_i) B_j \\ &= E_i + \rho_i \sum_j F_{ij} B_j \end{aligned}$$

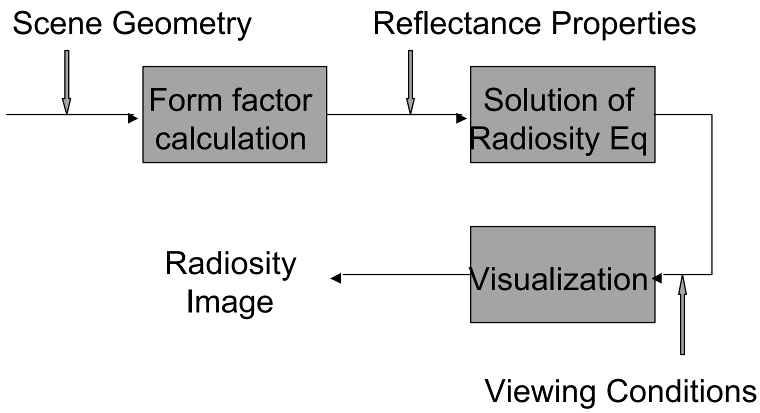
Radiosity as a Linear System

- Restate radiosity equation $B_i - \rho_i \sum_j F_{ij} B_j = E_i$
- In matrix form

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & \rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & \rho_2 F_{2n} \\ \vdots & \vdots & & \vdots \\ -\rho_n F_{n1} & \rho_n F_{n2} & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

- Known: reflectances ρ_i, form factors F_i, emissions E_i
- Unknown: Radiosities B_i
- n linear equations in n unknowns

Radiosity “Pipeline”



Visualization

- Radiosity solution is viewer independent
- Can exploit graphics hardware to obtain image
- Convert color on patch to vertex color
- Easy part of radiosity method

Outline

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

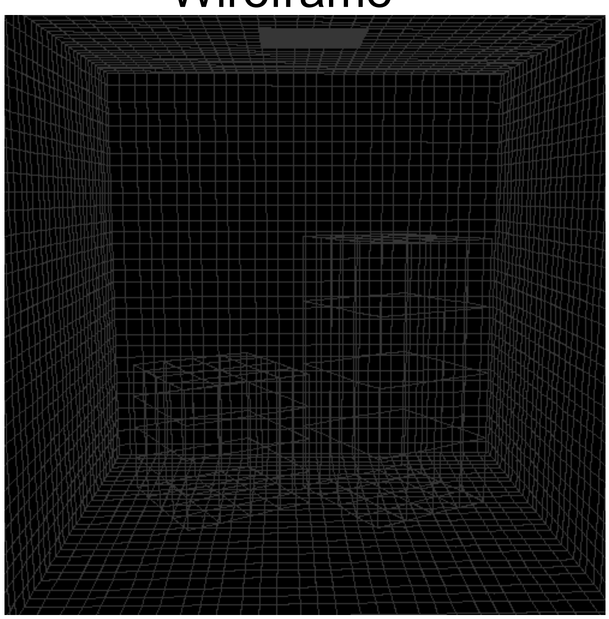
Computing Form Factors

- Visibility critical
- Two principal methods
  - Hemicube: exploit z-buffer hardware
  - Ray casting (can be slow)
  - Both exhibit aliasing effects
- For inter-visible elements
  - Many special cases can be solved analytically
  - Avoid full numeric approximation of double integral

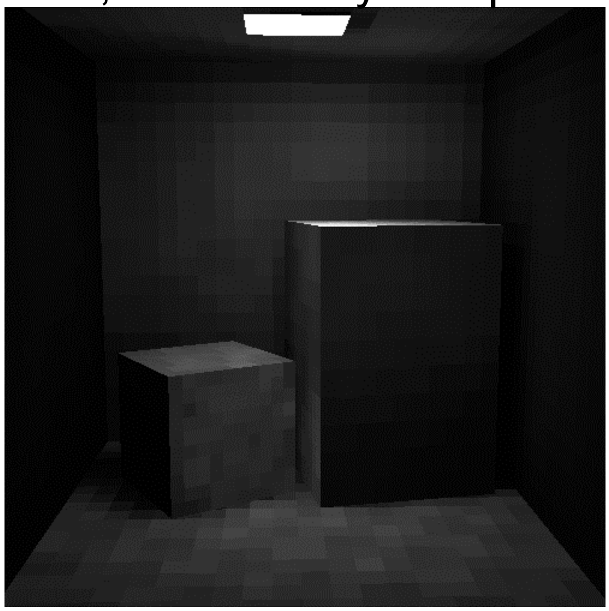
Hemicube Algorithm

- Render model onto a hemicube as seen from the center of a patch
- Store patch identifiers j instead of color
- Use z-buffer to resolve visibility
- Efficiently implementable in hardware
- Examples of antialiasing [Chandran et al.]

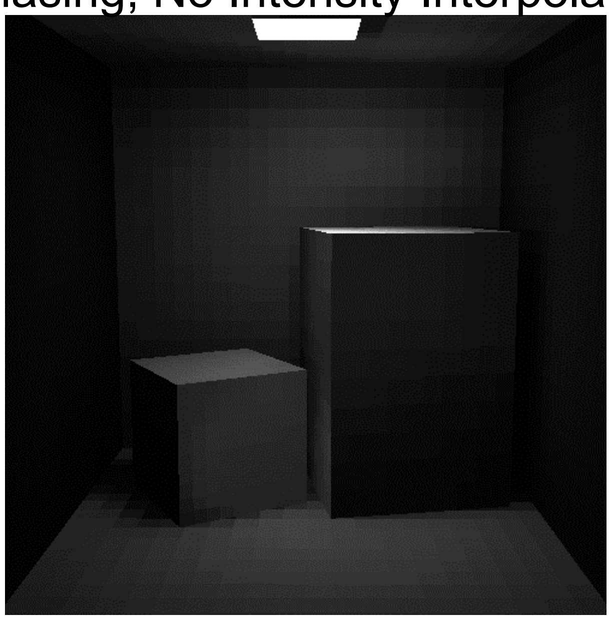
Wireframe



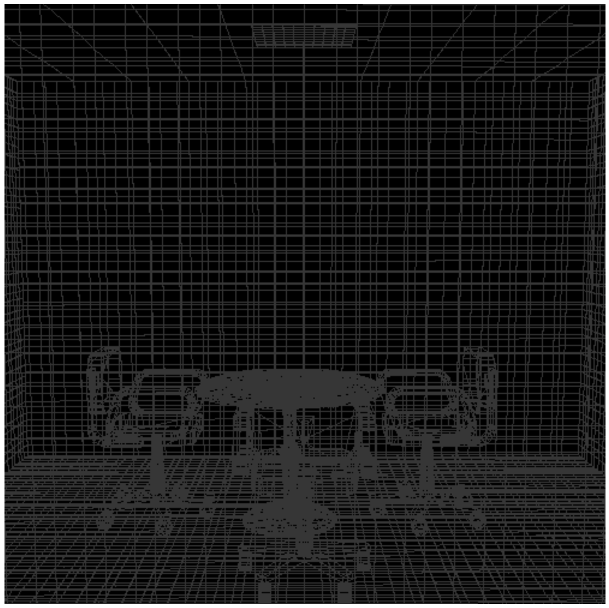
Classical, No Intensity Interpolation



Antialiasing, No Intensity Interpolation



Wireframe



Classical, Resolution 300

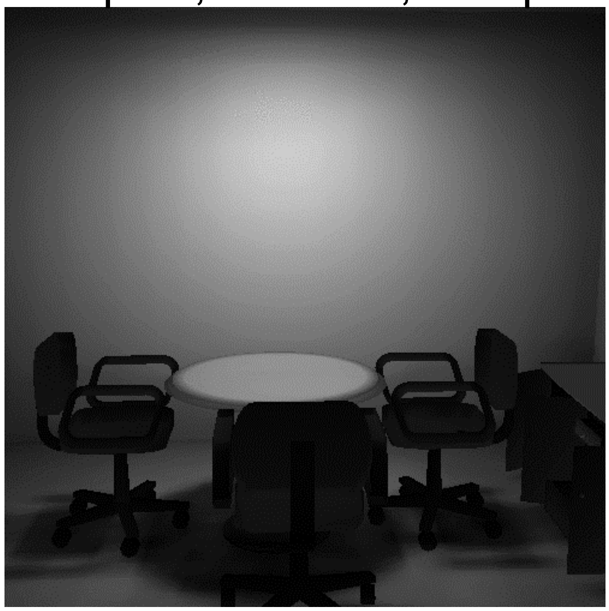
Classical, Resolution 1200

Classical, Resolution 2500

Supersampling, Resolution 100

Classical, Resolution 2500, Interpolated

Supersampled, Res 100, Interpolated



Outline

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

Radiosity Equation Revisited

- Direct form

$$B_i \;=\; E_i + \rho_i \sum_j F_{ij} B_j$$

- As matrix equation

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & \rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & \rho_2 F_{2n} \\ \vdots & \vdots & & \vdots \\ -\rho_n F_{n1} & \rho_n F_{n2} & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

- Unknown: radiosity B_i
- Known: emission E_i, form factor $F_{i\,j}$, reflect. ρ_i

Classical Radiosity Algorithms

- Matrix Radiosity
  - Diagonally dominant matrix
  - Use Gauss-Seidel iterative solution
  - Time and space complexity is $O(n^2)$ for n elements
  - Memory cost excessive
- Progressive Refinement Radiosity
  - Solve equations incrementally with form factors
  - Time complexity is $O(n \cdot s)$ for s iterations
  - Used more commonly (space complexity $O(n)$)

Matrix Radiosity

- Compute all form factors F_{ij}
- Make initial approximation to radiosity
  - Emitting elements $B_i = E_i$
  - Other elements $B_i = 0$
- Apply equation to get next approximation

$$B_i' = E_i + \rho_i \sum_j F_{ij} B_j$$

- Iterate with new approximation
- Intuitively
  - Gather incoming light for each element i
  - Base new estimate on previous estimate

Radiosity Summary

- Assumptions
  - Opaque Lambertian surfaces (ideal diffuse)
  - Radiosity constant across each element
- Radiosity computation structure
  - Break scene into patches
  - Compute form factors between patches
    - Lighting independent
  - Solve linear radiosity equation
    - Viewer independent
  - Render using standard hardware

Lecture Summary

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

Preview

- Next Lecture
  - Radiosity refinements
  - Combining ray tracing and radiosity
- Assignment 7 (Ray Tracer) due April 24
- Different from OpenGL programming (150 pts)