15-462 Computer Graphics I
Lecture 7

Lighting and Shading

Light Sources
Phong Illumination Model
Normal Vectors
[Angel, Ch. 6.1-6.4]

February 4, 2003
Frank Pfenning
Carnegie Mellon University

http://www.cs.cmu.edu/~fp/courses/graphics/

Remarks About Assignment 2

- Remember that object transformations are applied in the reverse order in which they appear in the code!
- Remember that transformation matrices are multiplied on the right and executed from right to left: (R S T)v = R (S (T v))!
- Look at the model solution (when it is out) and make sure you understand it before the midterm

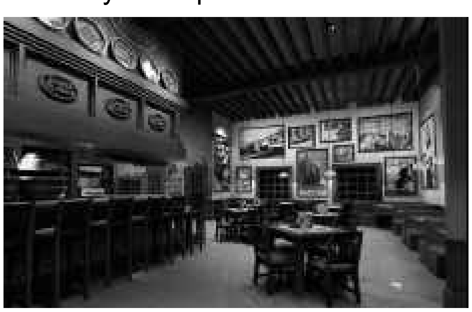
Outline

- Light Sources
- Phong Illumination Model
- Normal Vectors

Lighting and Shading

- Approximate physical reality
- Ray tracing:
  - Follow light rays through a scene
  - Accurate, but expensive (off-line)
- Radiosity:
  - Calculate surface inter-reflection approximately
  - Accurate, especially interiors, but expensive (off-line)
- Phong Illumination model (this lecture):
  - Approximate only interaction light, surface, viewer
  - Relatively fast (on-line), supported in OpenGL

Radiosity Example



Restaurant Interior. Guillermo Leal, Evolucion Visual

Raytracing Example

Martin Moeck,
Siemens Lighting

Light Sources and Material Properties

- Appearance depends on
  - Light sources, their locations and properties
  - Material (surface) properties
  - Viewer position
- Ray tracing: from viewer into scene
- Radiosity: between surface patches
- Phong Model: at material, from light to viewer

Types of Light Sources

- Ambient light: no identifiable source or direction
- Point source: given only by point
- Distant light: given only by direction
- Spotlight: from source in direction
  - Cut-off angle defines a cone of light
  - Attenuation function (brighter in center)
- Light source described by a luminance
  - Each color is described separately
  - $I = [I_r \ I_g \ I_b]^T$ (I for intensity)
  - Sometimes calculate generically (applies to r, g, b)

Ambient Light

- Global ambient light
  - Independent of light source
  - Lights entire scene
- Local ambient light
  - Contributed by additional light sources
  - Can be different for each light and primary color
- Computationally inexpensive

$$\mathbf{I}_a = \begin{bmatrix} I_{ar} \\ I_{ag} \\ I_{ab} \end{bmatrix}$$

Point Source

- Given by a point p_0
- Light emitted equally in all directions

$$\mathbf{I}(\mathbf{p}_0) = \begin{bmatrix} I_r(\mathbf{p}_0) \\ I_g(\mathbf{p}_0) \\ I_b(\mathbf{p}_0) \end{bmatrix}$$

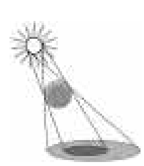
- Intensity decreases with square of distance

$$\mathbf{I}(\mathbf{p}, \mathbf{p}_0) = \frac{1}{|\mathbf{p} - \mathbf{p}_0|^2}\mathbf{I}(\mathbf{p}_0)$$

Limitations of Point Sources

- Shading and shadows inaccurate
- Example: penumbra (partial “soft” shadow)
- Similar problems with highlights
- Compensate with attenuation

$$\frac{1}{(a + bd + cd^2)}$$ d = distance $|p - p_0|$
a, b, c constants



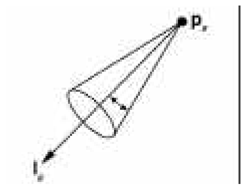
- Softens lighting
- Better with ray tracing
- Better with radiosity

Distant Light Source

- Given by a vector v
- Simplifies some calculations
- In OpenGL:
  - Point source $[x \ y \ z \ 1]^T$
  - Distant source $[x \ y \ z \ 0]^T$

Spotlight

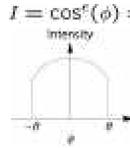
- Most complex light source in OpenGL
- Light still emanates from point
- Cut-off by cone determined by angle θ



Spotlight Attenuation

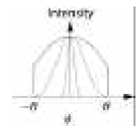
- Spotlight is brightest along l_s
- Vector v with angle ϕ from p to point on surface
- Intensity determined by cos ϕ
- Corresponds to projection of v onto l_s
- Spotlight exponent e determines rate

$$I = \cos^e(\phi) = (\mathbf{v} \cdot \mathbf{l}_s)^e$$



for e = 1

for e > 1
curve narrows



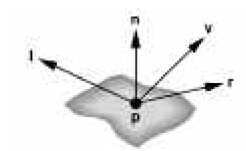
Outline

- Light Sources
- Phong Illumination Model
- Normal Vectors

Phong Illumination Model

- Calculate color for arbitrary point on surface
- Compromise between realism and efficiency
- Local computation (no visibility calculations)
- Basic inputs are material properties and l, n, v:

l = vector to light source
n = surface normal
v = vector to viewer
r = reflection of l at p
(determined by l and n)



Basic Calculation

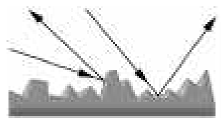
- Calculate each primary color separately
- Start with global ambient light
- Add reflections from each light source
- Clamp to [0, 1]
- Reflection decomposed into
  - Ambient reflection
  - Diffuse reflection
  - Specular reflection
- Based on ambient, diffuse, and specular lighting and material properties

Ambient Reflection

- Intensity of ambient light uniform at every point
- Ambient reflection coefficient k_a, $0 \leq k_a \leq 1$
- May be different for every surface and r,g,b
- Determines reflected fraction of ambient light
- L_a = ambient component of light source
- Ambient intensity $I_a = k_a L_a$
- Note: L_a is not a physically meaningful quantity

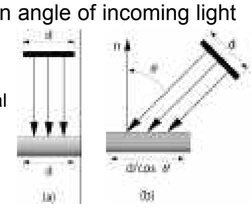
Diffuse Reflection

- Diffuse reflector scatters light
- Assume equally all direction
- Called Lambertian surface
- Diffuse reflection coefficient k_d, $0 \leq k_d \leq 1$
- Angle of incoming light still critical



Lambert's Law

- Intensity depends on angle of incoming light
- Recall
  - l = unit vector to light
  - n = unit surface normal
  - θ = angle to normal
- $\cos \theta = l \cdot n$
- $I_d = k_n (l \cdot n) L_d$
- With attenuation:

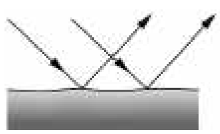


$$I_d = \frac{k_d}{a + bq + cq^2}(\mathbf{l} \cdot \mathbf{n}) L_d$$

q = distance to light source,
L_d = diffuse component of light

Specular Reflection

- Specular reflection coefficient k_s, $0 \leq k_s \leq 1$
- Shiny surfaces have high specular coefficient
- Used to model specular highlights
- Do not get mirror effect (need other techniques)

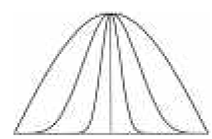


specular reflection

specular highlights

Shininess Coefficient

- L_s is specular component of light
- r is vector of perfect reflection of l about n
- v is vector to viewer
- ϕ is angle between v and r
- $I_s = k_s L_s \cos^\alpha \phi$
- α is shininess coefficient
- Compute $\cos \phi = r \cdot v$
- Requires $|r| = |v| = 1$
- Multiply distance term



Higher α is narrower

Summary of Phong Model

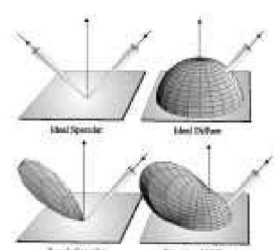
- Light components for each color:
  - Ambient (L_a), diffuse (L_d), specular (L_s)
- Material coefficients for each color:
  - Ambient (k_a), diffuse (k_d), specular (k_s)
- Distance q for surface point from light source

$$I = \frac{1}{a + bq + cq^2}(k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^\alpha) + k_a L_a$$

l = vector from light
n = surface normal
r = l reflected about n
v = vector to viewer

BRDF

- Bidirectional Reflection Distribution Function
- Measure for materials
- Isotropic vs. anisotropic
- Mathematically complex
- Programmable pixel shading?



Outline

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Normal Vectors

- Summarize Phong

$$I = \frac{1}{a + bq + cq^2}(k_d L_d (\mathbf{l} \cdot \mathbf{n}) + k_s L_s (\mathbf{r} \cdot \mathbf{v})^{\alpha}) + k_a L_a$$

- Surface normal n is critical
  - Calculate l · n
  - Calculate r and then r · v
- Must calculate and specify the normal vector
  - Even in OpenGL!
- Two examples: plane and sphere

Normals of a Plane, Method I

- Method I: given by ax + by + cz + d = 0
- Let p_0 be a known point on the plane
- Let p be an arbitrary point on the plane
- Recall: u · v = 0 iff u orthogonal v
- $n \cdot (p - p_0) = n \cdot p - n \cdot p_0 = 0$
- Consequently $n_0 = [a\ \ b\ \ c\ \ 0]^T$
- Normalize to $n = n_0/|n_0|$

Normals of a Plane, Method II

- Method II: plane given by p_0, p_1, p_2
- Points must not be collinear
- Recall: $u \times v$ orthogonal to u and v
- $n_0 = (p_1 - p_0) \times (p_2 - p_0)$
- Order of cross product determines orientation
- Normalize to $n = n_0/|n_0|$

Normals of Sphere

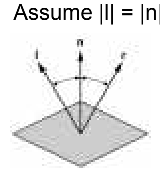
- Implicit Equation $f(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$
- Vector form: $f(p) = p \cdot p - 1 = 0$
- Normal given by gradient vector

$$\mathbf{n}_0 = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix} = 2\mathbf{p}$$

- Normalize $n_0/|n_0| = 2p/2 = p$

Angle of Reflection

- Perfect reflection: angle of incident equals angle of reflection
- Also: l, n, and r lie in the same plane
- Assume |l| = |n| = 1, guarantee |r| = 1



$$\mathbf{l} \cdot \mathbf{n} = \cos\theta = \mathbf{n} \cdot \mathbf{r}$$

$$\mathbf{r} = \alpha \mathbf{l} + \beta \mathbf{n}$$

Solution: $\alpha = -1$ and $\beta = 2\ (l \cdot n)$

$$\mathbf{r} = 2(\mathbf{l} \cdot \mathbf{n})\mathbf{n} - \mathbf{l}$$

Perhaps easier geometrically

Summary: Normal Vectors

- Critical for Phong model (diffuse and specular)
- Must calculate accurately (even in OpenGL)
- Pitfalls
  - Not unit length
  - How to set at surface boundary?
- Omitted
  - Refraction of transmitted light (Snell’s law)
  - Halfway vector (yet another optimization)

Summary

- Light Sources
- Phong Illumination Model
- Normal Vectors

Preview

- Polygonal shading
- Lighting and shading in OpenGL

Announcements

- Assignment 2 due Thursday
- Assignment 3 out Thursday, due in 2 weeks