15-462 Computer Graphics I
Lecture 18

Radiosity

Measures of Illumination
The Radiosity Equation
Form Factors
Radiosity Algorithms
[Angel, Ch 13.4-13.5]

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http://www.cs.cmu.edu/~fp/courses/graphics/

Local vs. Global Illumination

- Local illumination: Phong model (OpenGL)
  - Light to surface to viewer
  - No shadows, interreflections
  - Fast enough for interactive graphics
- Global illumination: Ray tracing
  - Multiple specular reflections and transmissions
  - Only one step of diffuse reflection
- Global illumination: Radiosity
  - All diffuse interreflections; shadows
  - Advanced: combine with specular reflection

Image vs. Object Space

- Image space: Ray tracing
  - Trace backwards from viewer
  - View-dependent calculation
  - Result: rasterized image (pixel by pixel)
- Object space: Radiosity
  - Assume only diffuse-diffuse interactions
  - View-independent calculation
  - Result: 3D model, color for each surface patch
  - Can render with OpenGL

Classical Radiosity Method

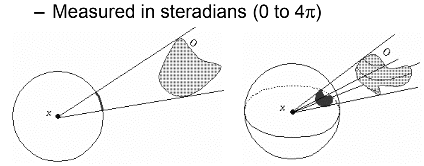
- Divide surfaces into patches (elements)
- Model light transfer between patches as system of linear equations
- Important assumptions:
  - Reflection and emission are diffuse
    - Recall: diffuse reflection is equal in all directions
    - So radiance is independent of direction
  - No participating media (no fog)
  - No transmission (only opaque surfaces)
  - Radiosity is constant across each element
  - Solve for R, G, B separately

Outline

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

Solid Angle

- 2D angle subtended by object O from point x:
  - Length of projection onto unit circle at x
  - Measured in radians (0 to 2π)
- 3D solid angle subtended by O from point x:
  - Area of of projection onto unit sphere at x
  - Measured in steradians (0 to 4π)



J. Stewart

Radiant Power and Radiosity

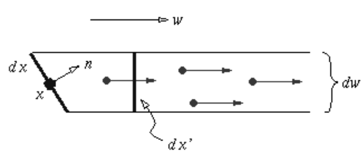
- Radiant power P
  - Rate at which light energy is transmitted
  - Dimension: power = energy / time
- Flux density Φ
  - Radiant power per unit area of the surface
  - Dimension: power / area
- Irradiance E: incident flux density of surface
- Radiosity B: exitant flux density of surface
  - Dimension: power / area
- Flux density at a point $\Phi(x) = dP/dx$

Power at Point in a Direction

- Radiant intensity I
  - Power radiated per unit solid angle by point source
  - Dimension: power / solid angle
- Radiant intensity in direction ω
  - $I(\omega) = dP/d\omega$
- Radiance $L(x, \omega)$
  - Flux density at point x in direction ω
  - Dimension: power / (area $\times$ solid angle)

Radiance

- Measured across surface in direction ω



J. Stewart '98

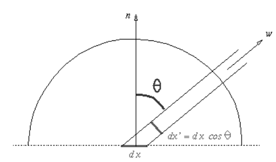
- For angle θ between ω and normal **n**

$$L(x,\omega) = \frac{d^2P}{d\omega\, dx'} = \frac{d^2P}{d\omega\, \cos\theta\, dx}$$

Radiosity and Radiance

- Radiosity $B(x) = dP / dx$
- Radiance $L(x,\omega) = d^2P / d\omega \cos\theta\, dx$
- Let Ω be set of all directions above x

$$B(x) = \int_{\Omega} L(x,\omega) \cos\theta\, d\omega$$



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Balance of Energy

- Lambertian surfaces (ideal diffuse reflector)
- Divided into n elements
- Variables
  - A_i Area of element i (computable)
  - B_i Radiosity of element i (unknown)
  - E_i Radiant emitted flux density of element i (given)
  - ρ_i Reflectance of element i (given)
  - $F_{j\,i}$ Form factor from j to i (computable)

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^{n} F_{ji} A_j B_j$$

Form Factors

- Form factor $F_{i\,j}$: Fraction of light leaving element i arriving at element j
- Depends on
  - Shape of patches i and j
  - Relative orientation of both patches
  - Distance between patches
  - Occlusion by other patches

Form Factor Equation

- Polar angles θ and θ' between normals and ray between x and y
- Visibility function v(x,y) = 0 if ray from x to y is occluded, v(x,y) = 1 otherwise
- Distance r between x and y

$$A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos\theta \ \cos\theta'}{\pi r^2} v(x,y)\, dy\, dx$$

Reciprocity

- Symmetry of form factor

$$A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos\theta \ \cos\theta'}{\pi r^2} v(x,y)\, dy\, dx = A_j F_{ji}$$

- Divide earlier radiosity equation

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^{n} F_{ji} A_j B_j$$

by A_i

$$\begin{aligned} B_i &= E_i + \rho_i \sum_j (F_{ji} A_j / A_i) B_j \\ &= E_i + \rho_i \sum_j F_{ij} B_j \end{aligned}$$

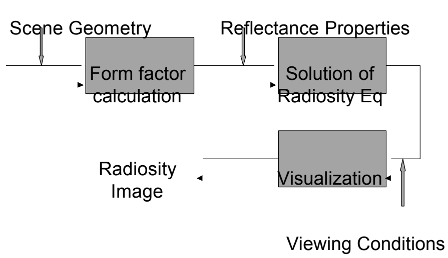
Radiosity as a Linear System

- Restate radiosity equation $B_i - \rho_i \sum_j F_{ij} B_j = E_i$
- In matrix form

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & \rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & \rho_2 F_{2n} \\ \vdots & \vdots & & \vdots \\ -\rho_n F_{n1} & \rho_n F_{n2} & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

- Known: reflectances ρ_i, form factors F_i, emissions E_i
- Unknown: Radiosities B_i
- n linear equations in n unknowns

Radiosity "Pipeline"



Visualization

- Radiosity solution is viewer independent
- Can exploit graphics hardware to obtain image
- Convert color on patch to vertex color
- Easy part of radiosity method

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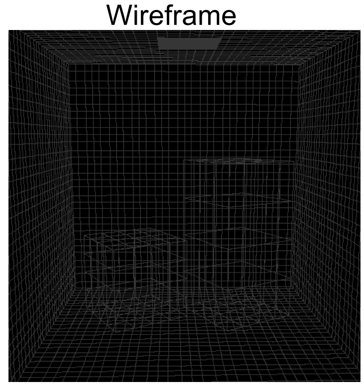
Computing Form Factors

- Visibility critical
- Two principal methods
  - Hemicube: exploit z-buffer hardware
  - Ray casting (can be slow)
  - Both exhibit aliasing effects
- For inter-visible elements
  - Many special cases can be solved analytically
  - Avoid full numeric approximation of double integral

Hemicube Algorithm

- Render model onto a hemicube as seen from the center of a patch
- Store patch identifiers j instead of color
- Use z-buffer to resolve visibility
- Efficiently implementable in hardware
- Examples of antialiasing [Chandran et al.]

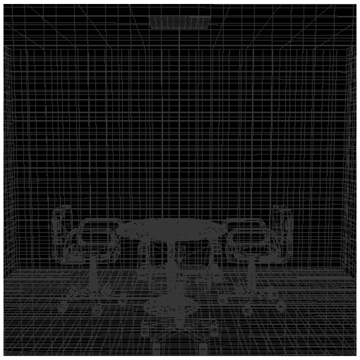
Wireframe



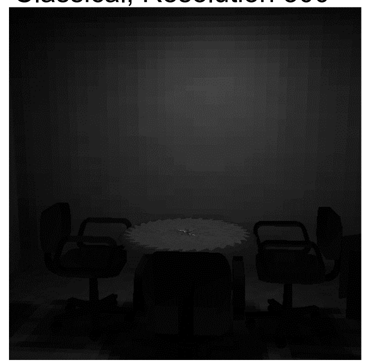
Classical, No Intensity Interpolation

Antialiasing, No Intensity Interpolation

Wireframe



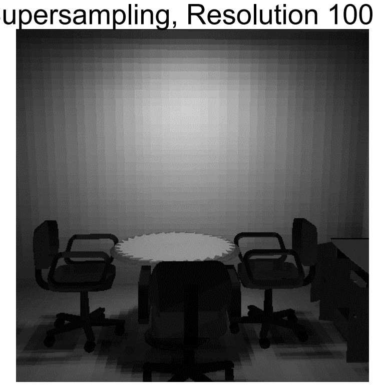
Classical, Resolution 300



Classical, Resolution 1200

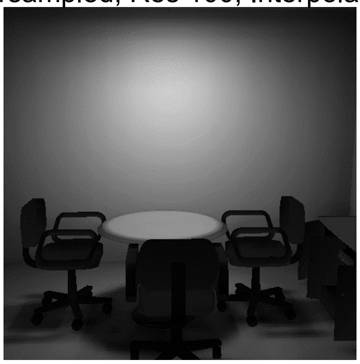
Classical, Resolution 2500

Supersampling, Resolution 100



Classical, Resolution 2500, Interpolated

Supersampled, Res 100, Interpolated



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Radiosity Equation Revisited

- Direct form

$$B_i = E_i + \rho_i \sum_j F_{ij} B_j$$

- As matrix equation

$$\begin{bmatrix} 1-\rho_1F_{11} & -\rho_1F_{12} & \cdots & \rho_1F_{1n} \\ -\rho_2F_{21} & 1-\rho_2F_{22} & \cdots & \rho_2F_{2n} \\ \vdots & \vdots & & \vdots \\ -\rho_nF_{n1} & \rho_nF_{n2} & \cdots & 1-\rho_nF_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

- Unknown: radiosity B_i
- Known: emission E_i, form factor $F_{i\,j}$, reflect. ρ_i

Classical Radiosity Algorithms

- Matrix Radiosity
  - Diagonally dominant matrix
  - Use Gauss-Seidel iterative solution
  - Time and space complexity is $O(n^2)$ for n elements
  - Memory cost excessive
- Progressive Refinement Radiosity
  - Solve equations incrementally with form factors
  - Time complexity is $O(n \cdot s)$ for s iterations
  - Used more commonly (space complexity O(n))

Matrix Radiosity

- Compute all form factors $F_{i\,j}$
- Make initial approximation to radiosity
  - Emitting elements $B_i = E_i$
  - Other elements $B_i = 0$
- Apply equation to get next approximation

$$B_i' = E_i + \rho_i \sum_j F_{ij} B_j$$

- Iterate with new approximation
- Intuitively
  - Gather incoming light for each element i
  - Base new estimate on previous estimate

Radiosity Summary

- Assumptions
  - Opaque Lambertian surfaces (ideal diffuse)
  - Radiosity constant across each element
- Radiosity computation structure
  - Break scene into patches
  - Compute form factors between patches
    - Lighting independent
  - Solve linear radiosity equation
    - Viewer independent
  - Render using standard hardware

Lecture Summary

- Measures of Illumination
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Preview

- Next Lecture
  - Radiosity refinements
  - Combining ray tracing and radiosity
- Assignment 7 (Ray Tracer) due April 24
- Different from OpenGL programming (150 pts)