



## Correction to: Multi-server queueing systems with multiple priority classes

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### Correction to: Queueing Syst <https://doi.org/10.1007/s11134-005-2898-7>

We correct the expressions of the matrix  $\mathbf{B}^{(\ell)}$  on page 340 and the matrix  $\mathbf{L}^{(\ell)}$  on page 341 in [1]. Specifically, the following are the corrected expressions of these matrices:

$$\mathbf{B}^{(\ell)} = \mu_L \begin{pmatrix} \min(2, \ell) & & & \\ & 1 & & \\ & & 1 & \\ & & & \mathbf{0} \end{pmatrix} \quad (1)$$

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$$\mathbf{L}^{(\ell)} = \left( \begin{array}{c|c|c|c|c|c|c}
 \begin{matrix} -\sigma_1 & \lambda_M & \lambda_H \\ \mu_M & -\sigma_2 & \\ \mu_H & & -\sigma_3 \end{matrix} & & & & & & \\
 \hline
 \mathbf{t}^{(1)} & \mathbf{T}^{(1)} & & & & & \\
 \hline
 & \mathbf{t}^{(2)} & \mathbf{T}^{(2)} & & & & \\
 \hline
 \mathbf{t}^{(3)} & & & \mathbf{T}^{(3)} & & & \\
 \hline
 & \mathbf{t}^{(4)} & & & \mathbf{T}^{(4)} & & \\
 \hline
 \mathbf{t}^{(5)} & & & & & \mathbf{T}^{(5)} & \\
 \hline
 & \mathbf{t}^{(6)} & & & & & \mathbf{T}^{(6)}
 \end{array} \right) \tag{2}$$

for all  $\ell \geq 0$ , where the definitions of the notation in the matrices are unchanged from [1] except the zero matrix  $\mathbf{0}$  in (1), whose size needs to be corrected to  $12 \times 12$ .

These matrices represent the transitions shown in the left panel of Figure 3 in [1]. The transition from state  $(1H, 0M, uL)$  to  $(1H, 0M, (u - 1)L)$ , namely the third diagonal element of  $\mathbf{B}^{(\ell)}$ , was missing in the original expression. The transition rates from  $(0H, 1M, uL)$  to two states labeled with  $(1H, 1M, uL)$ , namely the  $(2, k)$  element of  $\mathbf{L}^{(\ell)}$  for  $8 \leq k \leq 11$ , are  $\lambda_H \mathbf{p}^{(MH, M)}$  and  $\lambda_H \mathbf{p}^{(MH, H)}$ , but erroneously were  $\lambda_M \mathbf{p}^{(MH, M)}$  and  $\lambda_M \mathbf{p}^{(MH, H)}$  in the original expression. Likewise, the transition rates from  $(1H, 0M, uL)$  to  $(1H, 1M, uL)$ , namely the  $(3, k)$  element of  $\mathbf{L}^{(\ell)}$  for  $8 \leq k \leq 11$ , are  $\lambda_M \mathbf{p}^{(MH, M)}$  and  $\lambda_M \mathbf{p}^{(MH, H)}$ , but were  $\lambda_H \mathbf{p}^{(MH, M)}$  and  $\lambda_H \mathbf{p}^{(MH, H)}$  in the original expression.

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## Reference

1. Harchol-Balter, M., Osogami, T., Scheller-Wolf, A., Wierman, A.: Multi-server queueing systems with multiple priority classes. *Queueing Syst.* **51**, 331–360 (2005). <https://doi.org/10.1007/s11134-005-2898-7>

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