Learning to Generate Fast Signal Processing Implementations

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Shorter version to be presented at ICML-2001

Overview

- Background and Motivation
- Key Signal Processing Observations
- Predicting Leaf Cache Misses
- Generating Fast Formulas
- Conclusions

Signal Processing

Many signal processing algorithms:

- take as input a signal X as a vector
- produce transformation of signal $Y = A\,X$

Issue:

- Naïve implementation of matrix multiplication is slow

Example signal processing applications:

- Real time audio, image, speech processing
- Analysis of large data sets

Factoring Signal Transforms

- Transformation matrices are highly structured
- Can factor transformation matrices
- Factorizations allow for faster implementations

Walsh-Hadamard Transform (WHT)

Highly structured, for example:

$$WHT(2^2) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Factorization or break down rule:

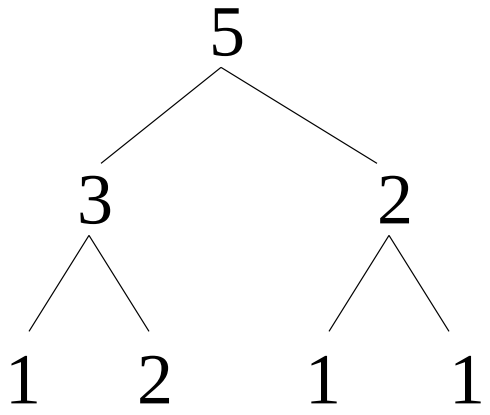
$$WHT(2^n) = \prod_{i=1}^{t} (I_{2^{n_1+\cdots+n_{i-1}}} \otimes WHT(2^{n_i}) \otimes I_{2^{n_{i+1}+\cdots+n_t}})$$

for positive integers n_i such that $n = n_1 + \cdots + n_t$

WHT Example

$$
\begin{aligned}
WHT(2^5) &= [WHT(2^3) \otimes I_{2^2}][I_{2^3} \otimes WHT(2^2)] \\
&= [\{(WHT(2^1) \otimes I_{2^2})(I_{2^1} \otimes WHT(2^2))\} \otimes I_{2^2}] \\
&\quad [I_{2^3} \otimes \{(WHT(2^1) \otimes I_{2^1})(I_{2^1} \otimes WHT(2^1))\}]
\end{aligned}
$$

We can visualize this as a split tree:



1-1 correspondence between split trees and formulas

Search Space

Large number of factorizations:

- $WHT(2^n)$ has $\Theta((4+\sqrt{8})^n/n^{3/2})$ different split trees
- $WHT(2^n)$ has $\Theta(5^n/n^{3/2})$ different binary split trees
- $WHT(2^{10})$ has 51,819 binary split trees

Varying Performance

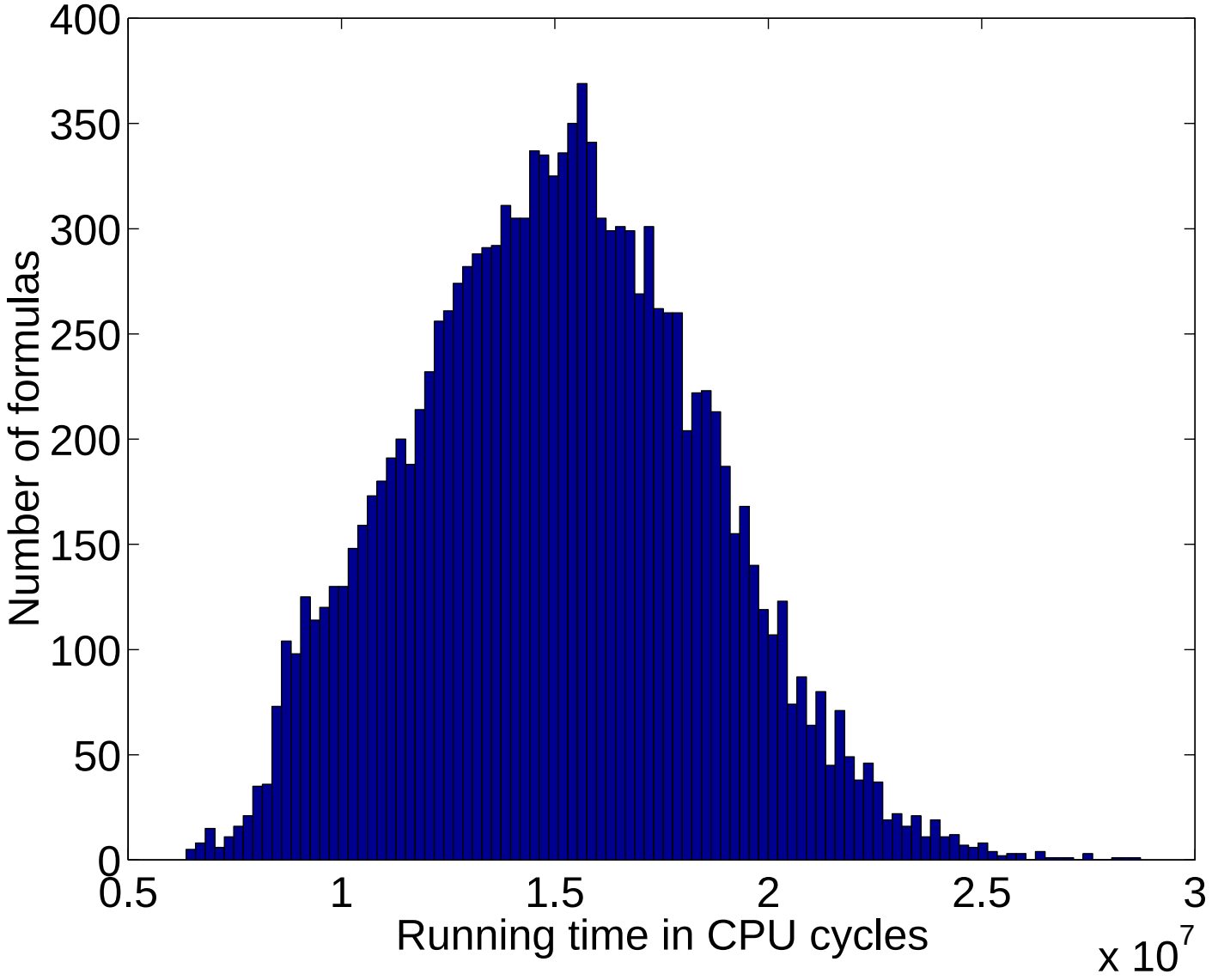
Varying performance of factorizations:

- Formulas have *very different* running times
- Small changes in the split tree can lead to significantly different running times
- Optimal formulas across machines are different

Reasons:

- Cache performance
- Utilization of execution units
- Number of registers

Histogram of $WHT(2^{16})$ Running Times



Problem

Huge search space of formulas

Want to find the fastest formula

- For a given transform
- For a given size
- For a given machine
- But for any input vector

Our Approach: Learn to generate fast formulas

- Learn to predict cache misses for leaves
- Use this as the base cases for determining values of different splittings
- Construct fast formulas by choosing best splittings

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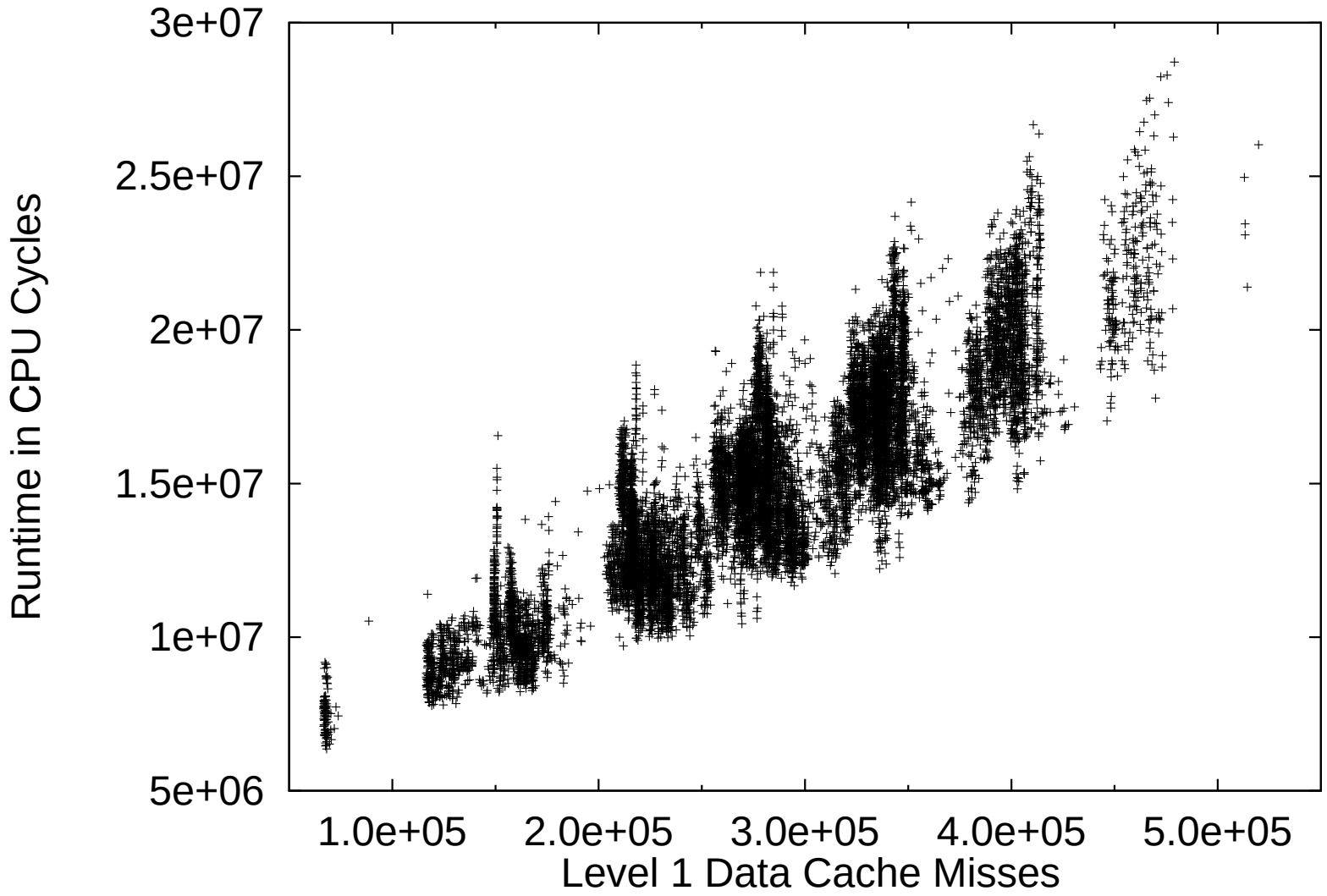
Run Times and Cache Misses



Run Times and Cache Misses

- Fastest formula has minimal number of cache misses

- Minimizing cache misses produces small group of formulas which contains the fastest formula

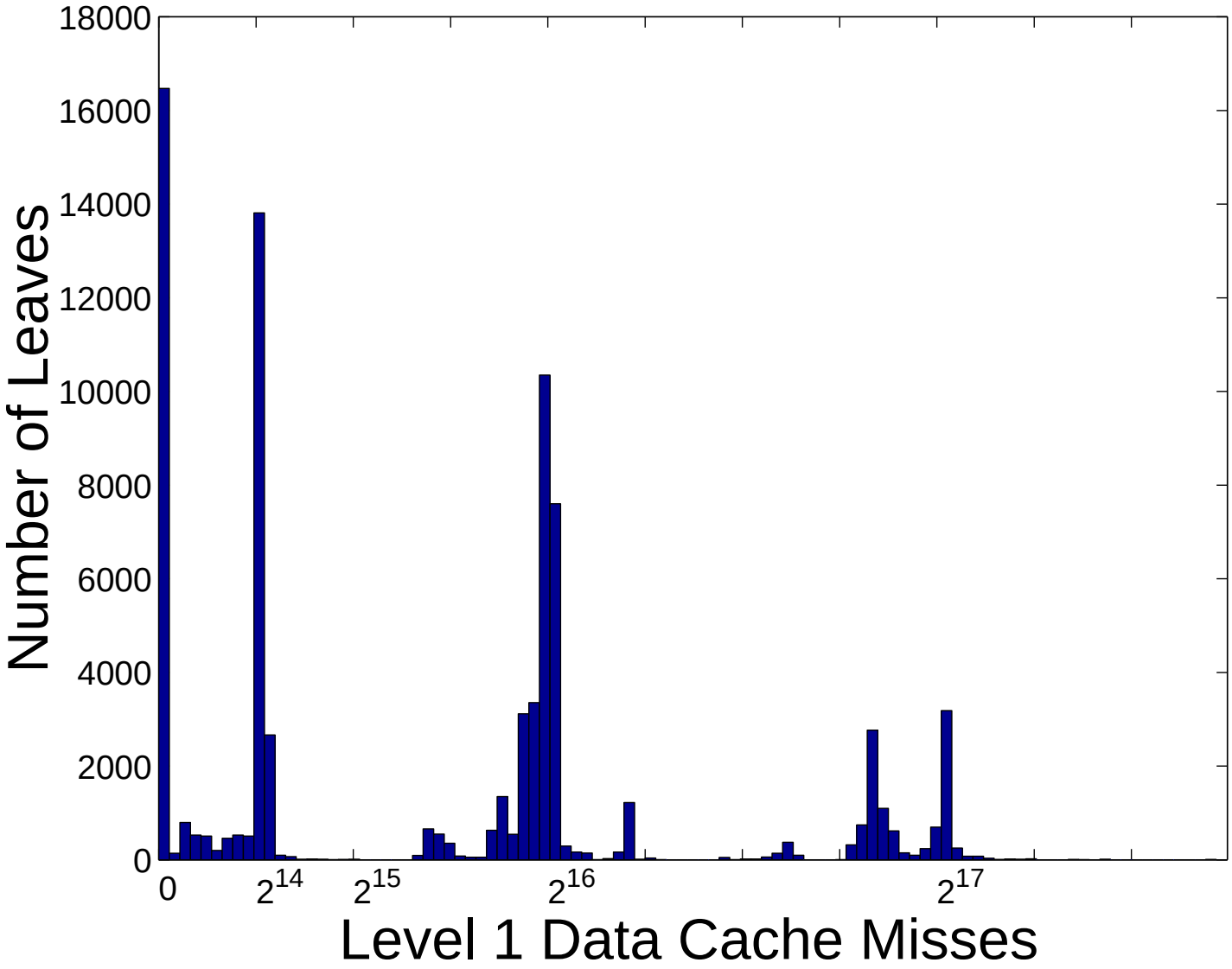
WHT Leaves

- WHT leaves are implemented as unrolled code (sizes 2^1 to 2^8)
- Internal nodes recursively call their children

- All run time and cache misses occur in the leaves
- Total run time or cache misses of a formula is the sum of that incurred by the leaves

- If we can predict for leaves,
  then we can predict for entire formulas

Leaf Cache Misses: $WHT(2^{16})$ example



Leaf Cache Misses

- The number of cache misses incurred by leaves is only of a few possible values
- These values are fractions of the transform size

- We can predict one of a few categories instead of real valued number of cache misses
- We can learn across different sizes by learning the categories corresponding to fractions of the transform size

Review of Observations

- Fastest formula has minimal number of cache misses
- All computation performed in the leaves
- Leaf cache misses only have a few values
- Leaf cache misses are fractions of transform size

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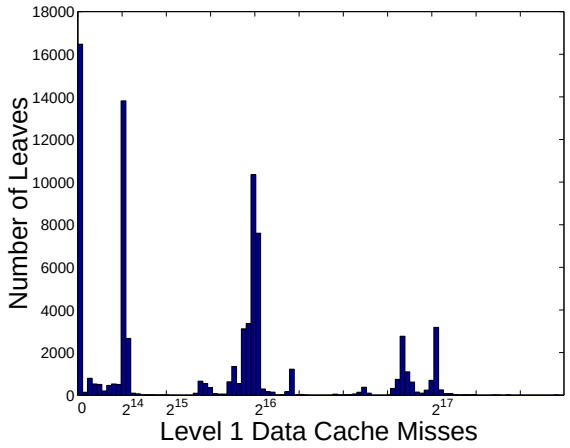
Predicting Leaf Cache Misses

- Want to learn to accurately predict leaf cache misses
- Should then be able to predict cache misses for entire formulas

Learning Algorithm

1. Collect cache misses for leaves of WHT formulas

2. Classify (cache misses / transform size) as:
   - near-zero if less than 1/8
   - near-quarter if less than 1/2
   - near-whole if less than 3/2
   - large otherwise.



3. Train a classification algorithm to predict one of the four classes given a leaf

Features for WHT Leaves

Need to describe WHT leaves with features

Could use:

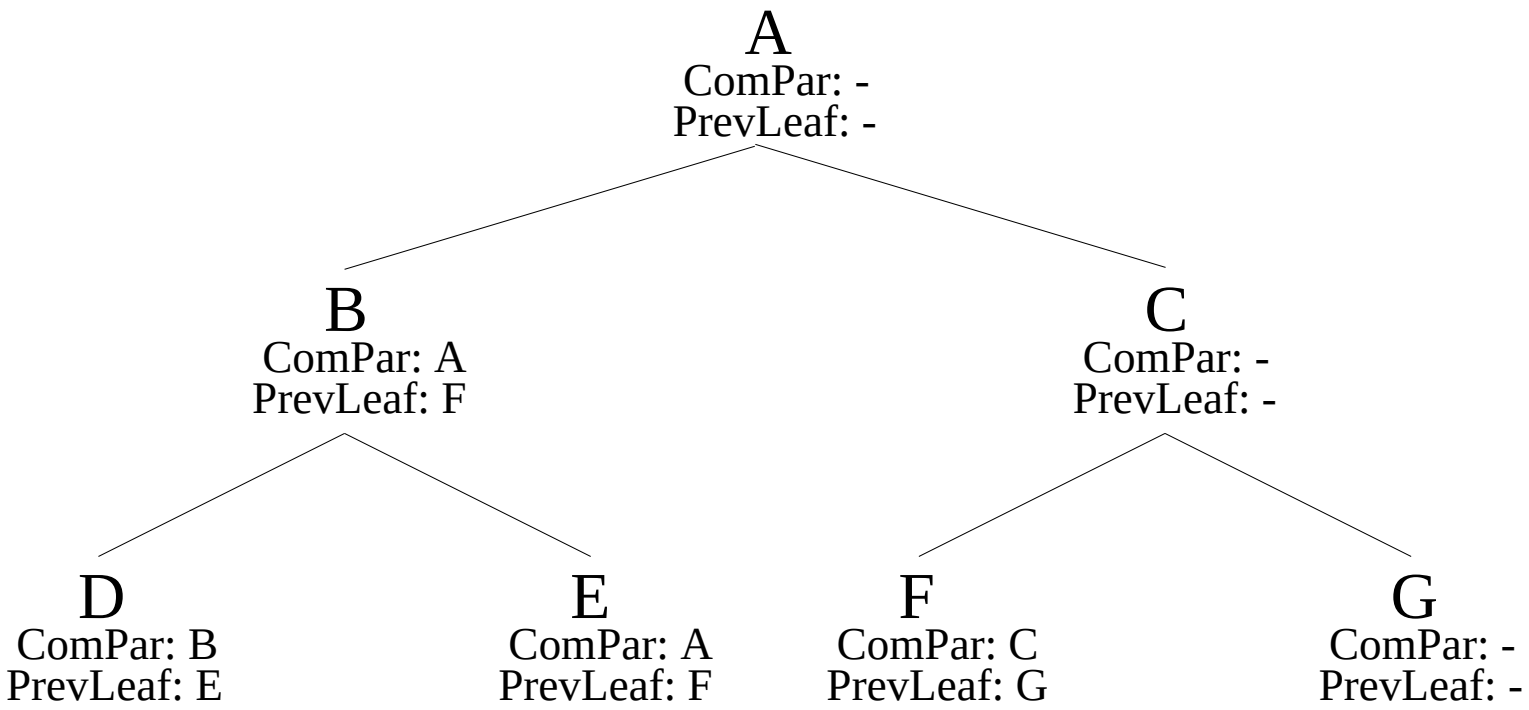
- Size of the given leaf
- Stride of the given leaf

Stride:

- Determines how a node accesses its input and output data
- Greatly impacts cache performance
- Determined by location of node in split tree

More Features for WHT Leaves

- Size and stride of the given leaf
- Size and stride of the parent of the given leaf
- Size and stride of the common parent



Review: Learning Algorithm

1. Collect cache misses for leaves of WHT formulas
2. Classify (cache misses / transform size) as:
   - near-zero if less than 1/8
   - near-quarter if less than 1/2
   - near-whole if less than 3/2
   - large otherwise.
3. Describe leaves with features
4. Train a classification algorithm to predict one of the four classes given features for a leaf

Evaluation

- Trained a decision tree
- Used a random 10% of leaves of all binary $WHT(2^{16})$ split trees with no leaves of size 2^1
- Evaluated performance using subsets of formulas known to be fastest
- Can not evaluate over all formulas because there are too many possible formulas

Leaf Cache Miss Category Performance

Error rates for predicting cache miss category incurred by leaves

Binary No-2^1-Leaf

Size	Errors
2^{12}	0.5%
2^{13}	1.7%
2^{14}	0.9%
2^{15}	0.9%
2^{16}	0.7%

Binary No-2^1-Leaf Rightmost

Size	Errors
2^{17}	1.7%
2^{18}	1.7%
2^{19}	1.7%
2^{20}	1.6%
2^{21}	1.6%

Trained on one size, performs well across many!

Predicting Cache Misses for Entire Formulas

Average percentage error for predicting cache misses for entire formulas

Binary No-2^1-Leaf

Size	Errors
2^{12}	12.7%
2^{13}	8.6%
2^{14}	6.7%
2^{15}	5.2%
2^{16}	4.6%

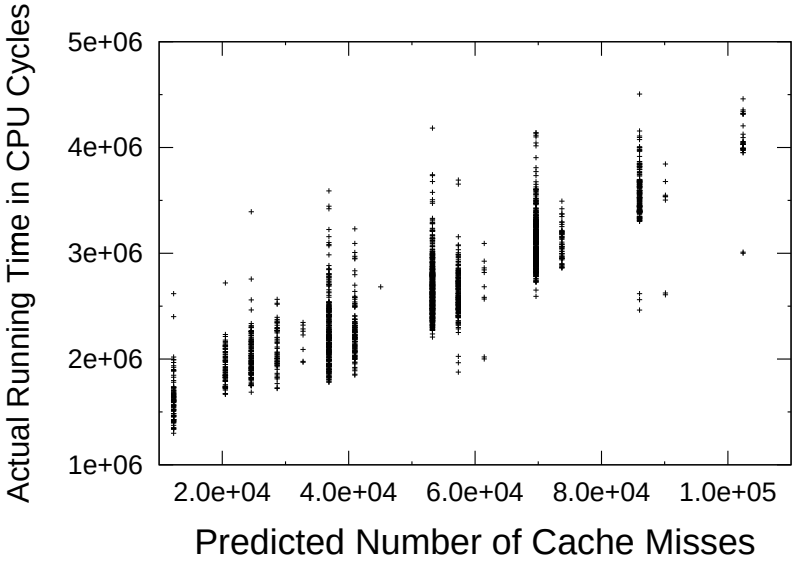
Binary No-2^1-Leaf Rightmost

Size	Errors
2^{17}	8.2%
2^{18}	8.2%
2^{19}	7.9%
2^{20}	8.1%
2^{21}	10.4%

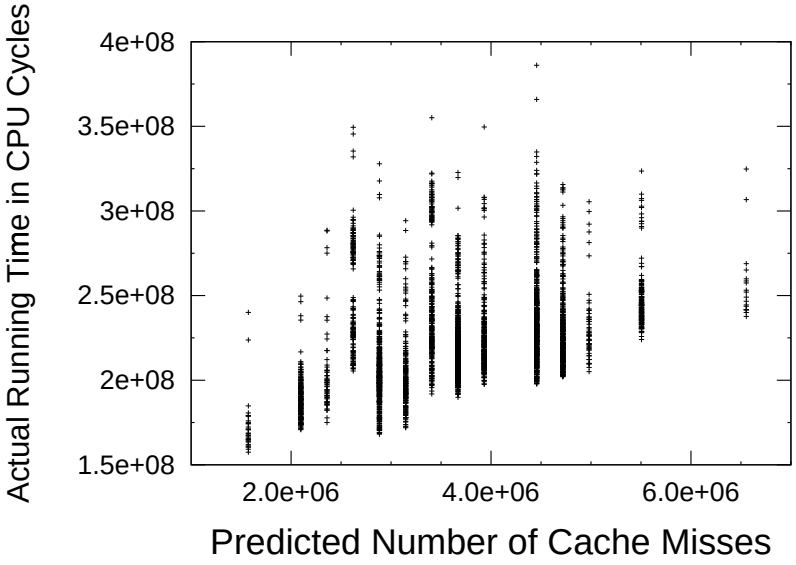
$Error = \frac{1}{|TestSet|} \sum_{i \in TestSet} \frac{|a_i - p_i|}{a_i}$, where a_i and p_i are the actual and predicted number of cache misses for formula i.

Runtime Versus Predicted Cache Misses

Binary No-2^1-Leaf
$WHT(2^{14})$



Binary No-2^1-Leaf
Rightmost $WHT(2^{20})$



Review: Predicting Cache Misses

By learning to predict leaf cache misses:

- Accurately predict cache misses for entire formulas
- Fastest formulas have fewest predicted cache misses
- Predict accurately across many transform sizes while trained on one size

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Generating Fast Formulas

- Can now quickly predict cache misses for a formula
- Fastest formulas have minimal cache misses
- But still MANY formulas to search through

Can we learn to generate fast formulas?

Generating Fast Formulas: Approach

Control Learning Problem:

- Learn to control the generation of formulas to produce fast ones

Want to grow the fastest WHT split tree:

- Begin with a root node of the desired size

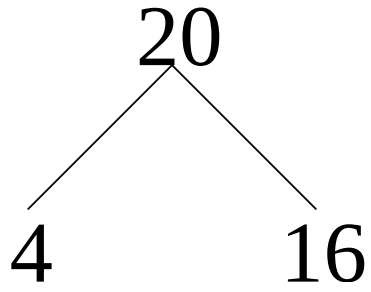
Generating Fast Formulas: Approach

Control Learning Problem:

- Learn to control the generation of formulas to produce fast ones

Want to grow the fastest WHT split tree:

- Begin with a root node of the desired size
- Grow best possible children



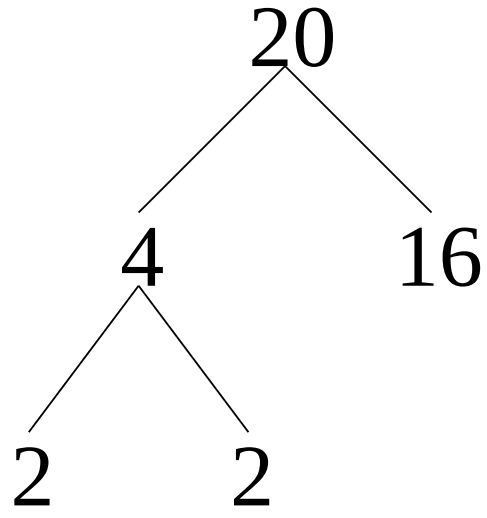
Generating Fast Formulas: Approach

Control Learning Problem:

- Learn to control the generation of formulas to produce fast ones

Want to grow the fastest WHT split tree:

- Begin with a root node of the desired size
- Grow best possible children
- Recurse on each of the children



Generating Fast Formulas: Approach

- Try to formulate in terms of Markov Decision Processes (MDPs) and Reinforcement Learning (RL)
- Final formulation not an MDP
- Final formulation borrows concepts from RL

MDPs

An MDP is a tuple $(\mathcal{S}, \mathcal{A}, T, C)$:

- $\mathcal{S}$ is a set of states
- $\mathcal{A}$ is a set of actions
- $T: \mathcal{S} \times \mathcal{A} \rightarrow \mathcal{S}$ is a transition function that maps the current state and action to the next state
- $C: \mathcal{S} \times \mathcal{A} \rightarrow \Re$ is a cost function that maps the current state and action onto its real valued cost

Markov Property: T and C only depend on the current state and action

MDPs and RL

Agent:

- Observes current state
- Selects action to take
- Receives the cost for that action in that state
- Observes next state, and repeat

Reinforcement learning provides methods for finding a policy $\pi: \mathcal{S} \rightarrow \mathcal{A}$ that selects the best action at each state that minimizes the sum of costs incurred

Basic Formulation

Given a size, want to grow a fast WHT split tree

- States = unexpanded nodes in split tree
- Start state = root node of given size w/ no children
- Actions = ways to split a node, giving it children

  OR, make the node a leaf
- Cost Function =
  - Zero when giving children to a node
  - The leaf's run time when making a node a leaf
- Goal = minimize sum of costs

Detail: State Space Representation

States = unexpanded nodes in split tree

But how to represent the states???

Modified leaf features for arbitrary nodes:

- Size and stride of the given node
- Size and stride of the parent of the given node
- Size and stride of the common parent to this node

Detail: Cost Function

Ideal Cost Function =

- Zero when giving children to a node
- The leaf's run time when making a node a leaf

But, a leaf's runtime is not easily obtained

However, we can predict cache misses for leaves!

Used Cost Function =

- Zero when giving children to a node
- The leaf's predicted cache misses when making a node a leaf

Now we really minimize the number of cache misses

Difficulty: Transition Function

What is the transition function for this problem?

Given that 2 children of the root are grown:

- Which node is the next state?
- When will we transition back to the sibling?
- Where to transition to from a leaf node?
- And still maintain the Markov property?

We depart from the MDP framework here . . .

Our Approach

Problem advantages:

- Deterministic and known actions
- Deterministic and known cost function

  (learned decision tree)

Approach:

- Define an optimal value function on states
- Run DP to determine value function

  (basically like solving an MDP)

Value Function

Define an optimal value function on states:

- Value of a state is the cost of the best subtree
- Value of root node is the cost of the best formula
- Choose children that have minimal sum of values

Mathematically: Value Function on States

State = unexpanded node in split tree,

described by 6 features

The optimal value of a state is:

$$V^*(state) = \min_{subtrees} \sum_{leaf \in subtree} CacheMisses(leaf)$$

- Min over all possible subtrees of the given state
- $CacheMisses()$ returns the predicted number of cache misses for the given leaf

Recursive Formulation of Value Function

Define:

$$LeafCM(state) = \begin{cases} CacheMisses(state), \text{ if state can be a leaf} \\ \infty, \text{ if state cannot be a leaf} \end{cases}$$

and

$$SplitV(state) = \min_{splittings} \sum_{child \in splitting} V^*(child)$$

Then:

$$V^*(state) = \min\{LeafCM(state), SplitV(state)\}$$

Computing the Value Function

Use dynamic programming to calculate value function:

- Consider all possible sets of children of the root
- Recursively call DP on each of the children, memoizing results
- Determine set of children with minimal sum of values
- Root's value is this minimal sum of values

Generating Fast Formulas

Generate split tree with minimal Value (or near minimal)

- Consider all possible sets of children of the root
- Choose those that have the minimal sum of values
- Recurse on children

Evaluation

Difficulty:

- Do not know what the optimal formula is
- Too many formulas to exhaust at larger sizes

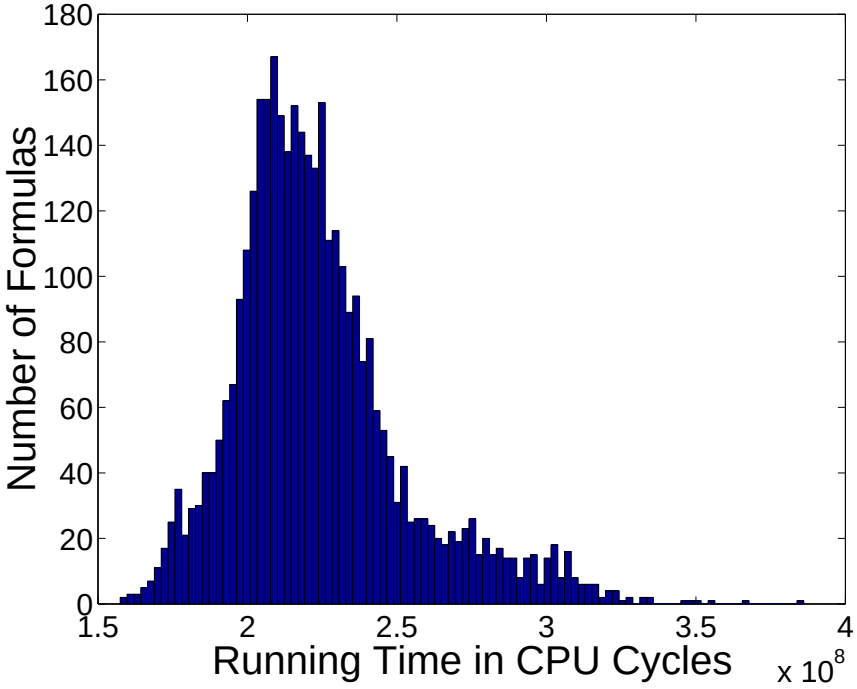
Possible:

- Exhaust over limited subspaces
- Limit based on signal processing knowledge and prior experience from using different search methods
- Compare my method with best found by this limited exhaust

Fast Formula Generation Results

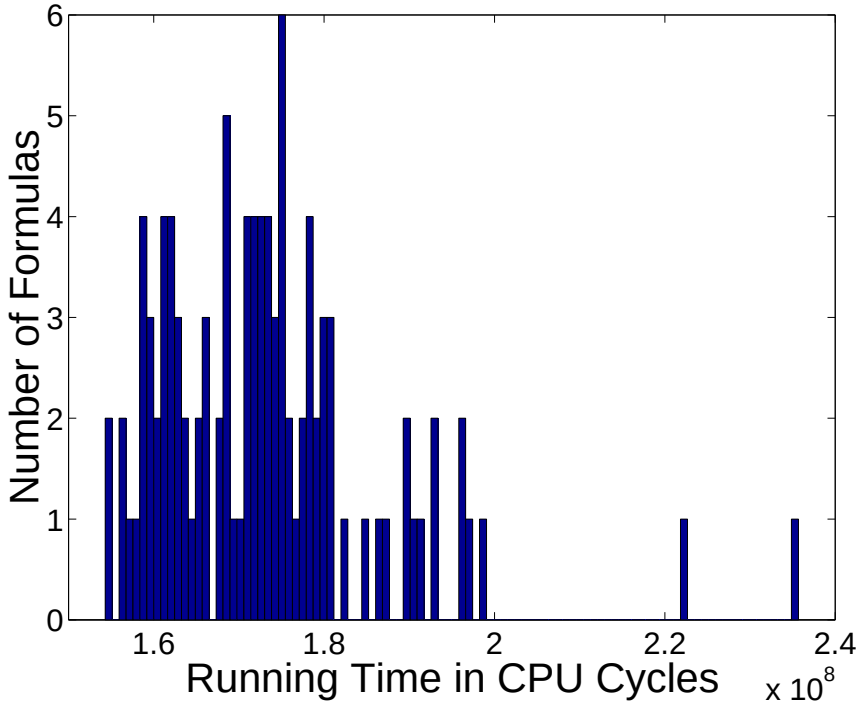
Size	Number of Formulas Generated	Generated Included the Fastest Known	# of Fastest Formulas in Generated
2^{12}	101	yes	77
2^{13}	86	yes	4
2^{14}	101	yes	70
2^{15}	86	yes	11
2^{16}	101	yes	68
2^{17}	86	yes	15
2^{18}	101	yes	25
2^{19}	86	yes	16
2^{20}	101	yes	16

Histograms: $WHT(2^{20})$



Limited Exhaust

Binary No-2^1-Leaf Rightmost



Our method

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Conclusions

- New method for constructing fast WHT formulas
- Generates fastest known formulas!
- Method can be trained on data for one size and perform well across many sizes
- Also, can learn to accurately predict cache misses of formulas

On going and future work:

- Test and extend to other architectures
- Extend to other transforms

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Questions?