Graduate Algorithms	Manuel Blum, Virginia Vassilevska, Chris Wang
Problem Set 3 (Last Modified Date: February 25, 2005) Due: March 4th, 2005 by 11.59pm	

You can work in groups of up to 3. Please let us know, for each question, if you have seen the question before. You will be graded out of min(100, max possible) points for this assignment. Unless otherwise specified, n is the number of vertices of a graph. Please submit your solution in ALL of the following formats by email to `virgi+algo@cs.cmu.edu` and `chengwen+gradalg@cs.cmu.edu`: ps, pdf and tex. Please use 'Submission: names' in the subject line. If you use the result of any paper, please understand and rephrase its content.

1. [20pts] Prove or disprove: given 2 splay trees A and B, if you can perform a sequence of splay operations to convert A to B, then there is a sequence of splay operations to convert B back to A.

2. Give an efficient algorithm (and data structures) to 6-color a given planar graph G. Remember to prove your running time.

   ```
    2pts O(n^2)
    5pts O(n log n)
   10pts O(n)
   ```

3. Give an efficient algorithm (and data structures) to 5-color a given planar graph G. Remember to prove your running time.

   ```
    4pts O(n^2)
   10pts O(n log n)
   20pts O(n)
   ```

4. Given a planar graph G, find an efficient algorithm to find a vertex cut of size $O(\sqrt{n})$ which partitions G into two parts each of size at most $\frac{n}{2}$ vertices.

   ```
    5pts for polytime
   10pts O(n log n)
   20pts O(n)

   Or 20pts show it is NP-hard
   ```

5. Given a weighted undirected planar graph G, find a minimum spanning tree of G efficiently. ($\alpha(n, n)$ is *not* constant.)

   ```
    5pts for polytime
   10pts O(n log n)
   20pts O(n) randomized algorithm
   40pts O(n) deterministic algorithm
   ```