Algorithms in the Real World
Assignment #2 — Due Date: 11 Oct 02

Problem 1: (20pt) Suppose that a bipartite graph with n nodes on the left and n nodes on the right is constructed by connecting each node on the left to d randomly-selected nodes on the right (each chosen with probability $1/n$). All random choices are made independently, and there is no restriction on the degree of a node on the right, i.e., nodes on the right may have any degree from 0 to dn. Show that for any fixed $\beta > 1$, and any fixed $d > \beta + 1$, there exists a fixed α such that the graph has the (α, β) expansion property with probability > 0.

Problem 2: (20pt)

A. Prove that the bisection width (i.e. the number of edges that must be removed to separate a graph into two equal-sized parts, within 1) of the complete graph with n vertices is $(n/2)^2$.

B. Prove that the bisection width of the n-node hypercube is $n/2$. This should be proved from below and above. (Hint: show that the complete graph can be embedded in the hypercube so that vertices map to vertices, edges in the complete graph map to paths in the hypercube, and each hypercube ends up supporting the same number of paths.)

Problem 3: (20pt) In class, and in the Karypis and Kumar reading, we covered a multilevel edge-separator algorithm. In this problem you need to generalize this technique to work for vertex separators directly (do not use a postprocessing stage). In particular:

1. Argue why coarsening using a maximal matching is or is not still appropriate.
2. Describe what we should keep track of when coarsening (contracting) the graph (e.g. on the edge separator version each edge kept a weight representing the number of original edges between two multivertices).
3. Describe how we project the solution of the coarsened version back onto the original graph (the recursive solution must return a vertex separator).
4. Describe a variant of Kernighan-Lin or (preferably) the Fiduccia-Mattheyses heuristic for vertex separators. Be explicit about what the gain metric is.

Problem 4: (20pt) Prove that given a class of graphs satisfying an $O(n^{(d-1)/d})$ edge-separator theorem, all members must have bounded degree.

or (more challenging)

Prove that given a class of graphs satisfying an $O(n^{(d-1)/d})$ vertex-separator theorem, the edges of any member can be directed so the out-degree is bounded. You can use the fact mentioned in class that such graphs have bounded density (i.e. the average degree is bounded).

Problem 5: (20pt) Consider applying divide-and-conquer to graphs and lets say that merging the two recursive solutions takes $f(s)$ time, where s is the number of edges separating the two graphs. For each of the following $f(s)$, and assuming you are given an edge separator tree for which all separators for the subgraphs of size n are 1/3-2/3 balanced and bounded by $kn^{1/2}$, what is the running time of such an approach.

1. s
2. $s \log s$
3. s^2
4. s^4