15-451 Algorithms, Fall 2000

Homework # 6 **due: Wed-Thurs Nov 29,30, 2000**

Groundrules:

- This is an oral presentation assignment. As stated in the course information handout, you should work in groups of three. At some point before **Monday Nov 27 at midnight** you should sign up for a 1-hour time slot on the signup sheet on the course web page.
- You are not required to hand anything in at your presentation, but you may if you choose. If you do hand something in, it will be taken into consideration (in a non-negative way) in the grading.

Problems:

1. [Set-Cover]

   (a) The *set-cover* problem is the following: Given n points labeled $1, 2, \ldots, n$, and m subsets of these points $s_1, s_2, \ldots, s_m$, and an integer k, is it possible to cover all the points using only k of the sets s_i? (I.e., every point should be in at least one of these k sets). For instance, if $n = 6$ and the sets are $s_1 = \{1, 2, 3\}$, $s_2 = \{1, 4\}$, $s_3 = \{2, 5\}$, and $s_4 = \{3, 6\}$, then it is possible to cover all the points with three sets (namely, s_2, s_3, and s_4) but not with two of them.

   Prove that the set-cover problem is NP-complete by reducing from the vertex-cover problem. (Remember, in vertex-cover you are given a graph and want to know if there exist k vertices such that every edge has at least one of those vertices as an endpoint.) Also, say why set-cover is in NP.

   (b) The *fractional set cover* problem is like the set-cover problem, except instead of choosing or not choosing each set, you instead assign each set a fraction between 0 and 1. The requirement is that for each point i, if you add up the fractions assigned to sets that cover that point, you must get a total of ≥ 1. The goal is to minimize the sum of all the fractions.

   For example, if there are 3 points and the sets are $s_1 = \{1, 2\}$, $s_2 = \{2, 3\}$, and $s_3 = \{3, 1\}$, then the best fractional set cover is to assign each set the fraction $1/2$; this covers all the points and the total sum is $1/2 + 1/2 + 1/2 = 3/2$. In contrast, the best standard set cover requires 2 sets. (Note that a standard set cover is also a legal fractional cover, but not necessarily vice-versa.)

   Show how to solve the fractional set cover problem using *linear programming*. Be sure to specify what the variables are, what the constraints are, and what you are trying to minimize or maximize.

2. [Multicommodity Flow] The *multicommodity flow* problem is just like the standard network flow problem except we have p sources $s_1, \ldots, s_p$ and p sinks $t_1, \ldots, t_p$. The stuff flowing from s_1 has to go to t_1, the stuff from s_2 has to go to t_2, and so on. For each sink t_i we have a *demand* d_i. (E.g., we need to get d_1 trucks from s_1 to t_1, d_2 trucks from s_2 to t_2, and so on.) Our goal is to solve for a *feasible solution* — a solution satisfying the demands — if one exists.

   For example, in the graph from last year's quiz 2 (see the webpage), if we needed to get 5 units of flow from S to T, and 1 unit of flow from the top node to the bottom node, then there is a feasible solution. But if we needed to get 2 units from the top node to the bottom node, then there wouldn't be a feasible solution.

   (a) Show how to solve this using linear programming. I.e., reduce multicommodity flow to linear programming.

   (b) Suppose we didn't care which sink received flow from which source (all the trucks are carrying the same thing). Show how to reduce this to the standard max flow problem (i.e., how to construct a new graph such that solving for the max flow in the new graph gives you the answer you want).

3. Your choice of Problem 11.17 or 11.18 in Manber. Problem 11.17 is easier but 11.18 is fun if you like linear algebra.