15-853:Algorithms in the Real World

Data Compression: Lecture 2.5

Summary so far

Model generates probabilities, **Coder** uses them

Probabilities are related to **information**. The more you know, the less info a message will give.

More "skew" in probabilities gives lower **Entropy H** and therefore better compression

Context can help "skew" probabilities (lower H)

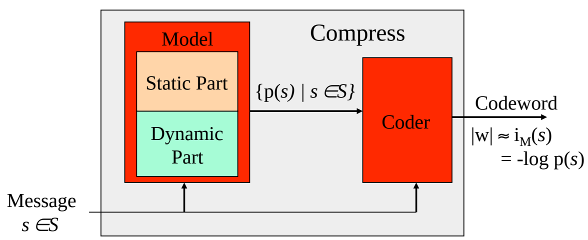
Average length l_a for **optimal prefix code** bound by

$$H \le l_a < H + 1$$

Huffman codes are optimal prefix codes

Arithmetic codes allow "blending" among messages

Encoding: Model and Coder



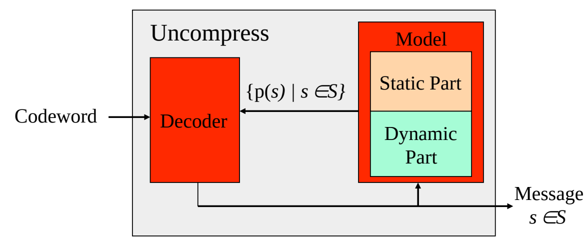
The **Static part** of the model is fixed

The **Dynamic part** is based on previous messages

The "optimality" of the code is relative to the probabilities.

If they are not accurate, the code is not going to be efficient

Decoding: Model and Decoder



The **probabilities** $\{p(s) \mid s \in S\}$ generated by the model need to be the same as generated in the encoder.

Note: consecutive "messages" can be from a different message sets, and the probability distribution can change

Codes with Dynamic Probabilities

Huffman codes:

Need to generate a new tree for new probabilities.

Small changes in probability, typically make small changes to the Huffman tree.

"**Adaptive Huffman codes**" update the tree without having to completely recalculate it.

Used frequently in practice

Arithmetic codes:

Need to recalculate the f(m) values based on current probabilities.

Can be done with a balanced tree.

Compression Outline

Introduction: Lossy vs. Lossless, Benchmarks, …

Information Theory: Entropy, etc.

Probability Coding: Huffman + Arithmetic Coding

Applications of Probability Coding: PPM + others

- Transform coding: move to front, run-length, …
- Context coding: fixed context, partial matching

Lempel-Ziv Algorithms: LZ77, gzip, compress, ...

Other Lossless Algorithms: Burrows-Wheeler

Lossy algorithms for images: JPEG, MPEG, ...

Compressing graphs and meshes: BBK

Applications of Probability Coding

How do we generate the probabilities?

Using character frequencies directly does not work very well (e.g. 4.5 bits/char for text).

Technique 1: transforming the data

- Run length coding (ITU Fax standard)
- Move-to-front coding (Used in Burrows-Wheeler)
- Residual coding (JPEG LS)

Technique 2: using conditional probabilities

- Fixed context (JBIG…almost)
- Partial matching (PPM)

Run Length Coding

Code by specifying message value followed by the number of repeated values:

e.g. **abbbaacccca => (a,1),(b,3),(a,2),(c,4),(a,1)**

The characters and counts can be coded based on frequency.

This allows for small number of bits overhead for low counts such as 1.

Facsimile ITU T4 (Group 3)

Standard used by all home Fax Machines

ITU = International Telecommunications Standard

Run length encodes sequences of black+white pixels

Fixed Huffman Code for all documents. e.g.

Run length	White	Black
1	000111	010
2	0111	11
10	00111	0000100

Since alternate black and white, no need for values.

Facsimile ITU T4 (Group 3)

Transform: (run length)

- **input** : binary string
- **output** : interleaving of run lengths of black and white pixels

Probabilities: (on the output of the transform)

Static probabilities of each run length based on large set of test documents.

Coding: Huffman coding

Move to Front Coding

Transforms message sequence into sequence of integers, that can then be probability coded

Takes advantage of **temporal locality**

Start with values in a total order: e.g.: [a,b,c,d,…]

For each message

- output the position in the order
- move to the front of the order.

e.g.: c => output: 3, new order: [c,a,b,d,e,…]
a => output: 2, new order: [a,c,b,d,e,…]

Probability code the output.

The hope is that there is a bias for small numbers.

BZIP

Transform 1: (Burrows Wheeler) – covered later

- **input** : character string (block)
- **output** : reordered character string

Transform 2: (move to front)

- **input** : character string
- **output** : MTF numbering

Transform 3: (run length)

- **input** : MTF numbering
- **output** : sequence of run lengths

Probabilities: (on run lengths)

Dynamic based on counts for each block.

Coding: Originally arithmetic, but changed to Huffman in bzip2 due to patent concerns

Residual Coding

Typically used for message values that represent some sort of amplitude:
e.g. gray-level in an image, or amplitude in audio.

Basic Idea: guess next value based on current context. Output difference between guess and actual value. Use probability code on the output.

Consider compressing a stock value over time.

JPEG-LS

JPEG Lossless (not to be confused with lossless JPEG)
Just completed standardization process.

Codes in Raster Order. Uses 4 pixels as context:

NW	N	NE
W	*	

Tries to guess value of * based on W, NW, N and NE.

Works in two stages

JPEG LS: Stage 1

Uses the following equation:

$$P = \begin{cases} \min(N,W) & \text{if } NW \geq \max(N,W) \\ \max(N,W) & \text{if } NW < \min(N,W) \\ N + W - NW & \text{otherwise} \end{cases}$$

Averages neighbors and captures edges. e.g.

40	3	*
40	3	

30	40	*
20	30	

3	3	*
40	40	

JPEG LS: Stage 2

Uses 3 gradients: W-NW, NW-N, N-NE

Classifies each into one of 9 categories.

This gives 9^3=729 contexts, of which only 365 are needed because of symmetry.

Each context has a bias term that is used to adjust the previous prediction

After correction, the residual between guessed and actual value is found and coded using a Golomb-like code. (Golomb codes are similar to Gamma codes)

JPEG LS

Transform: (residual)
- **input** : gray-level image (8 bits/pixel)
- **output** : difference from guess at each pixel

Probabilities: (on the differences)

Static probabilities based on golomb code --- something like $p(n) = c/n^2$.

Coding: Golomb code

Using Conditional Probabilities: PPM

Use previous *k* characters as the context.
- Makes use of conditional probabilities

Base probabilities on counts:
e.g. if seen **th** 12 times followed by **e** 7 times, then the conditional probability ***p(e|th)*** = 7/12.

Need to keep *k* small so that dictionary does not get too large (typically less than 8).

Note that 8-gram Entropy of English is about 2.3bits/char while PPM does as well as 1.7bits/char

PPM: Partial Matching

Problem: What do we do if we have not seen the context followed by the character before?
- Cannot code 0 probabilities!

The key idea of PPM is to reduce context size if previous match has not been seen.
- If character has not been seen before with current context of size 3, try context of size 2, and then context of size 1, and then no context

Keep statistics for each context size < *k*

PPM: Changing between context

How do we tell the decoder to use a smaller context?

Send an **escape** message. Each escape tells the decoder to reduce the size of the context by 1.

The escape can be viewed as special character, but needs to be assigned a probability.
- Different variants of PPM use different heuristics for the probability.

PPM: Example Contexts

Context	Counts	Context	Counts	Context	Counts
Empty	A = 4	A	C = 3	AC	B = 1
	B = 2		$ = 1		C = 2
	C = 5	B	A = 2		$ = 2
	$ = 3		$ = 1	BA	C = 1
		C	A = 1		$ = 1
			B = 2	CA	C = 1
			C = 2		$ = 1
			$ = 3	CB	A = 2
					$ = 1
				CC	A = 1
					B = 1
					$ = 2

String = ACCBACCACBA k = 2

PPM: Other important optimizations

If context has not been seen before, automatically escape (no need for an escape symbol since decoder knows previous contexts)

Can exclude certain possibilities when switching down a context. This can save 20% in final length!

It is critical to use arithmetic codes since the probabilities are small.