## Learning a finite state device



Scenario: You have a box with buttons and lights. Box has internal state.

- lights $=f($ current state $)$
- current state $=g$ (button, prev state)
E.g., physical device, robot in building, simple interactive agent, Rubik's cube*.

Goal: build a predictive model of device (e.g., state diagram) by pressing buttons and observing lights.

## An example w/o hidden state

2 actions: $a, b$.


Generic algorithm when lights = state:

- Build a model.
- While not done: find an unexplored edge and take it.

What if several states look the same?
state $\neq$ sensation

Another example


What about redundancy? OK to learn logically equivalent state diagram.

What about hard-to-reach states?

One solution: Assume we can propose a hypothesis and receive a counterexample if it's not good enough.
(This is natural if we are going to use our learned model for some other task, or if we have test data we can use to test our hypotheses.)

- Set of states $Q$. Start state $q_{0}$.
- Set of actions $A$.
- Transition function $\delta: Q \times A \rightarrow Q$.
- Observations: obs $(q)$, obs $\left(q, a_{1} a_{2} a_{3} \ldots\right)$.

What does it mean for 2 states to be different?
$q$ and $q^{\prime}$ are different if there is an action sequence $a$ such that

$$
o b s(q, a) \neq \operatorname{obs}\left(q^{\prime}, a\right)
$$

## Learn-DFA: Learning with a Reset

(assume can reset to $q_{0}$ whenever desired)

Idea: build DFA, where each state is "named" by:

1. A string that gets you there.
2. The results of several experiments from that state.

Use (2) to distinguish states that look the same.

- $S=$ set of strings $a$ s.t. we know all states $\delta\left(q_{0}, a\right)$, for $a \in S$ are different.
- $E=$ set of experiments used to distinguish them. (do $S$ then $E$ )

$S A$ means strings $a_{1} \ldots a_{n}$ where $a_{1} \ldots a_{n-1} \in S$ and $a_{n} \in A$.



## Algorithm

Begin with $S=\{\lambda\}, E=\{\lambda\}$.

1. Fill in transitions to make a hypothesis FSM.
2. While exists $s \in S A$ such that no $s^{\prime} \in S$ has $\operatorname{row}\left(s^{\prime}\right)=\operatorname{row}(s)$, add $s$ into $S$, and go to 1 .
3. Query for counterexample $z$.
4. Consider all splits of $z$ into $\left(p_{i}, s_{i}\right)$, and replace $p_{i}$ with its predicted equivalent $\alpha_{i} \in S$.
5. Find $\alpha_{i} r_{i}$ and $\alpha_{i+1} r_{i+1}$ that produce different observations.
6. Add $r_{i+1}$ as a new experiment into $E$.

At most $n$ EQs and $n^{2}(|A|+1)+n \log m$ MQs.

Next time: getting rid of the reset.

Variations: other represntations (e.g., Rubik's cube).

