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Hybrid Spectral Transform Diagrams *

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Abstract

We give a uniform algebraic framework for computing hybrid spectral transforms in an efficient manner. Based on properties of the Kronecker product, we derive a set of recursive equations, which leads naturally to an algorithm for computing such transforms efficiently. As a result, many applications of transforms like the Walsh transform and the Reed-Muller transform, which were previously impossible because of memory constraints, have now become feasible. The same set of recursive equations also gives a new way of explaining hybrid transform diagrams, an efficient data-structure for integer valued boolean functions.

1 Introduction

Spectral transformations of boolean functions [15, 18], like the Walsh or Reed-Muller transformations, have numerous applications in computer aided design, especially in the synthesis and testing of combinational circuits. Using a straightforward implementation, the complexity of computing these transformations grows rapidly in the number of variables of the boolean function. In the past, this has severly limited the usefulness of spectral techniques for industrial applications. New techniques for computing the spectrum of a boolean function using binary decision diagrams, have made it possible to compute concise representations for the transforms of functions with several hundred variables [10].

This has led to a proliferation of similar transforms including MTBDD [6], FDD [5], BMD [5], OKFDD [12, 13, 19], and others [18]. It is often difficult to understand, how these transforms are related. In this paper we consider a class of transforms, called *hybrid spectral transforms* [7, 8], which encompasses all of these transforms. Let F be a function, which maps boolean vectors of length n into some set D. The Q–spectrum of F is a linear transformation Qf of the vector representation $f = (F(0,\ldots,0) \quad \ldots \quad F(1,\ldots,1))^T$ of F. We consider only spectral transformations Q, which are constructed as *Kronecker products* of certain elementary matrices. When we want to emphasize a particular hybrid spectral transformation Q, we call it a (hybrid) Q–transformation. We provide a uniform algebraic framework for reasoning about such transformations. This framework is based entirely on properties of the Kronecker product. We derive a set of recursive equations, which lead naturally to an algorithm for evaluating products of the form Qv where Q is given as a Kronecker product, and v is a vector. Typically the *hybrid spectral transformation* is given by a matrix Q. This matrix maps a vector representation of the original function to the *Q–spectrum* of F, the vector representation for the *Q–transform*. We show how various spectral transforms like the Walsh transform and the Reed-Muller transform can be computed efficiently using this algorithm. The same set of recursive equations gives a new way of explaining hybrid transform diagrams, an efficient data-structure for representing the Q–transform for D–valued boolean functions. The methodology developed in this paper also yields a concise classification of all known applicable spectral transformation diagrams.

2 Binary tree indexing

Let $F : B^n \to D$ be a D–valued boolean function; assume that F is given in tabular form $\{(b, F(b)) \mid b \in B^n\}$. Since the vector $b = (b_0,\ldots,b_{n-1})$ is *encoded* by an integer $x \in \{0,\ldots,2^n-1\}$, the list of values of F can be represented conveniently as a vector indexed by *boolean sequences* of length n. A *boolean sequence* is either empty (denoted ε), or of the form $c0$ or $c1$, where c is a boolean sequence. Let v be a vector of dimension 2^n, and c a boolean sequence of length $|c| \le n-1$. Then, we use the following indexing scheme: $v|_c$ is defined such that

$$v|_\varepsilon = v\ , \quad \text{and} \quad v|_c = \begin{pmatrix} v|_{c0} \\ v|_{c1} \end{pmatrix}\ ,$$

where $v|_{c0}$ and $v|_{c1}$ have the same length. This scheme is called *binary tree indexing*.

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3 Efficient Kronecker products

The *Kronecker product* $A \otimes B$ for a $(k \times l)$–matrix A and a $(m \times n)$–matrix B is defined as the following $(km \times ln)$–matrix:

$$A \otimes B = \begin{pmatrix} a_{11}B & a_{12}B & \cdots & a_{1l}B \\ a_{21}B & a_{22}B & \cdots & a_{2l}B \\ \vdots & \vdots & & \vdots \\ a_{k1}B & a_{k2}B & \cdots & a_{kl}B \end{pmatrix} .$$

The Kronecker product is associative, but not commutative. The following identity relates the Kronecker product and ordinary matrix multiplication.

$$(A_1 \otimes B_1) \cdot (A_2 \otimes B_2) = A_1 \cdot A_2 \otimes B_1 \cdot B_2 \quad .$$

Next, we derive a recursive algorithm for computing $Q \cdot f$, where the matrix Q is given by $Q = \bigotimes_{i=0}^{n-1} Q_i$ and binary tree indexing is used for the vector f. The algorithm is efficient because it avoids the construction of the Kronecker product. First, we consider the case where the Q_i is a (2×2)–matrix. The elements of Q_i are denoted $(Q_i)_{j,k}$, where $j, k \in \{1, 2\}$. We assume, that the dimension of $f|_c$ is 2^{n-k}. The algorithm is based on the following recursive equation.
When $k = n - 1$:

$$\left(\bigotimes_{j=k}^{n-1} Q_j\right) \cdot f|_c = Q_{n-1} \cdot f|_c \quad ,$$

otherwise:

$$\left(\bigotimes_{j=k}^{n-1} Q_j\right) \cdot f|_c = (Q_k \otimes I_{n,k}) \cdot \begin{pmatrix} \left(\bigotimes_{j=k+1}^{n-1} Q_j\right) \cdot f|_{c0} \\ \left(\bigotimes_{j=k+1}^{n-1} Q_j\right) \cdot f|_{c1} \end{pmatrix}$$

$I_{n,k}$ is an identity matrix of the dimension 2^{n-k}. The proof of the recursion can be found in [11].

Using these equations, $Q \cdot f$ is computed recursively by setting $k = 0$ and $c = \varepsilon$. We illustrate how the algorithm works by the following example. Let

$$f = (1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1)^T \quad ,$$

and $Q = Q_0 \otimes Q_1 \otimes Q_2$ where each $Q_i = \left(\begin{smallmatrix} 1 & 1 \\ -1 & 1 \end{smallmatrix}\right)$. The computation of $Q \cdot f$ starts with the four 2–element subvectors $f|_{00}, f|_{01}, f|_{10}$ and $f|_{11}$, which are transformed separately by Q_2. The results of these transformations are then assembled into two 4–element vectors, which are in turn transformed by $Q_1 \otimes I_{3,1}$ into the result $Q \cdot f$. This process is illustrated in figure 1.

Of course, in any implementation of the algorithm, the Kronecker products $(Q_j \otimes I_{n,j})$ need not be constructed. $(Q_j \otimes I_{n,j}) \cdot v$ can be computed by applying the (2×2)–matrix Q_j to the *blocks* $v|_0$ and $v|_1$ of the vector v. This operation only involves computing linear combinations of the vectors $v|_0$ and $v|_1$ with the scalars $(Q_j)_{kl}$.

This algorithm is easily generalized to Q–transforms over $\mathbb{Z}_l$, where the Q–transform is a Kronecker product made up from $(l \times l)$–matrices Q_j, $j = 0 \ldots n-1$. The recursive equations, however, also work for $(m \times l)$–matrices Q_j. For details we must refer to [11]. Such generalized recursive equations are used below to evaluate the Q–transforms.

4 Hybrid spectral transforms

In the remainder of the paper, we will restrict our attention to functions that map boolean vectors to $D = \mathbb{Z}_2$ or $\mathbb{Z}$. Such functions $F : B^n \to D$ may be uniformly expressed using the so-called *minterm-representation*. Let $\cdot$ and $+$ be the standard multiplication and addition operations on $\mathbb{Z}_2$ and $\mathbb{Z}$. Also, let $c : \{0, \ldots, 2^n - 1\} \to B^n$ be the encoding for boolean sequences from 2. The *minterm* $m(b)$ for a vector $b \in B^n$ is defined as follows:

$$m(b) = \prod_{i=0}^{n-1} t_i \ , \quad \text{where} \quad t_i = \begin{cases} x_i & \text{if } b_i = 1 \\ \overline{x_i} & \text{if } b_i = 0 \end{cases}$$

The *minterm representation* of $F : B^n \to D$ is given by:

$$F(x_0, \ldots, x_{n-1}) = \sum_{i=0}^{2^n-1} m(c(i)) \cdot f_i \quad .$$

This sum can be regarded the scalar product $m^T f$ of the *minterm*-vector $m = \left(m(c(0)) \quad \ldots \quad m(c(2^n - 1))\right)^T$ and the vector f for F as given above. The minterm vector can be expressed as a Kronecker product: $m = \left(\bigotimes_{j=0}^{n-1} (\overline{x_i} \quad x_i)\right)^T$.

The concept of minterm representation is easily generalized to get other representations of the same function:

$$F = m^T f = m^T I f = (m^T Q^{-1})(Qf)$$

where I is the appropriate identity matrix, and Q is some non-singular matrix. The vector Qf is called the *Q–spectrum of F* with respect to the *spectral transformation matrix* Q. Since Q is non-singular, the spectrum of a function provides a *canonical form* for the function. In general, any nonsingular matrix Q can be used for this purpose. For boolean functions it is useful to restrict spectral transforms to Kronecker products of (2×2)–matrices over $\{0, 1, -1\}$. Generally, two important cases of the Q–transformation are distinguished:

$Q = \bigotimes_k Q_0$, the *homogeneous transformation* [10],

$Q = \bigotimes_k Q_k$, not all Q_k–s are equal: the *heterogeneous*, or *hybrid transformation* [7].

Choosing Kronecker products to define spectral transformation matrices for boolean functions has several advantages. First, using the recursive algorithm given in section 3, the spectrum of any given function can be computed efficiently without actually ever constructing the huge transformation matrix. Enhancing the representation of the vector from *binary tree indexing* to a BDD-representation improves efficiency considerably

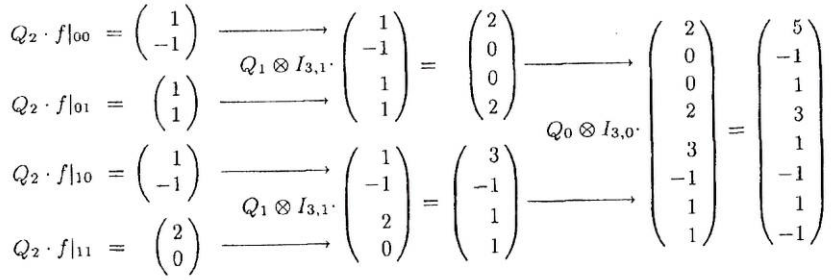

Figure 1: Computation of the spectrum

due to massive sharing of subvectors. Second, the utilization of Kronecker products of (2×2)–matrices in the transformations results in a modular representation of the function, which generalizes the notion of MTBDD to Q–transform diagrams.

5 Hybrid transform diagrams

The term-representation of F using a suitable Q–transform may be considerably shorter and more efficient to evaluate than its equivalent minterm representation. We develop BDD-like representations for Q–transforms, which allow efficient computation and evaluation of such transforms.

Let F be a D–valued boolean function in the variables $x_0, \ldots, x_{n-1}$, and $Q = \bigotimes_{i=0}^{n-1} Q_i$. Then, the Q–transform of F is given by $F(x_0, \ldots, x_{n-1}) = m^T Q^{-1} \tilde{f}$ where $\tilde{f} = Qf$ is the Q–spectrum of F. Inserting the definition of Q as well as the decomposition of m as mentioned earlier we obtain:

$$F(x_0, \ldots, x_{n-1}) = \left(\bigotimes_{j=0}^{n-1} \left(\begin{pmatrix} \overline{x_j} & x_j \end{pmatrix} \cdot Q_j^{-1} \right) \right) \cdot \tilde{f}$$

Now, for any given assignment to the variables, the value of the transform of F can be computed efficiently using a generalization of the recursive algorithm presented in section 3. Since $\begin{pmatrix} x_i & \overline{x_i} \end{pmatrix} \cdot Q_i^{-1}$ is always a (1×2)–matrix, the recursion becomes particularly simple.

However, in order to compute different values of the same transform, data-structures are needed to represent the transform efficiently. The key to this representation also stems from the recursive equation in section 3. Intuitively, this can be seen best by unfolding the recursion a few steps, as displayed in figure 2. Again, block operations are used:

$$\left(\begin{pmatrix} \overline{x_j} & x_j \end{pmatrix} \cdot Q_j^{-1} \right) \cdot v = l_j(x_j) \cdot v|_0 + r_j(x_j) \cdot v|_1$$

with scalar coefficients $l_j(x_j)$ and $r_j(x_j)$ (cf. [11]).

Using this recursive decomposition, evaluation of the Q–transform of F obviously follows a binary tree-pattern, so, the natural choice to represent the Q–transform is an annotated binary tree, which we call the *Q–transform tree*. The leaves of a Q–transform tree contain the elements of the Q–spectrum, whereas the branches on level j are labeled with scalar coefficients $l_j(x_j)$ and $r_j(x_j)$ (cf. [11]). An example best illustrates the situation. We compute the Q–transform tree for

$$f = (1 \quad -1 \quad 1 \quad -1 \quad 2 \quad -4 \quad 2 \quad -2)^T \ ,$$

where $Q = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. The Q–spectrum Qf can be computed as described in section 3: $Qf = (3 \quad 0 \quad 0 \quad 2 \quad -1 \quad 0 \quad 0 \quad -2)^T$. The evaluation of the Q–transform can be represented conveniently by the Q–transform tree, shown in figure 3.

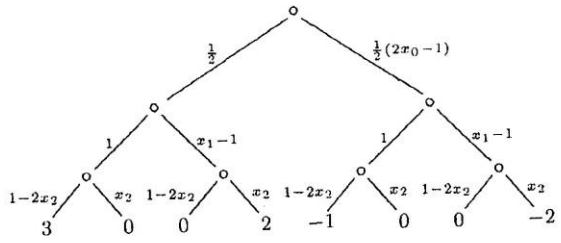

Figure 3: Q–transform tree for F

More formally, we define the Q–transform tree T for f, where the spectral transformation Q is given by a Kronecker product of (2×2)–matrices $Q = \bigotimes_{i=0}^{n-1} Q_i$, and the vector f is of dimension 2^n. For any $k \leq n-1$, and boolean sequence c of length less than $n-1$, the k, c–subtree $T_{k,c}$ of T is defined as follows.

- the left subtree of $T_{k,c}$ is $T_{k+1,c0}$, the branch leading to it is labeled with $l_k(x)$;
- the right subtree of $T_{k,c}$ is $T_{k+1,c1}$, the branch leading to it is labeled with $r_k(x)$;
- $T_{n,c0} = f|_{c0}$ and $T_{n,c1} = f|_{c1}$ are the leaves.

T itself is its own subtree $T_{0,\varepsilon}$.

Homogeneous *Shannon*–transform trees are an interesting special case. Since the Shannon transformation matrix is the (2×2)–identity matrix, the labeling on the branches becomes particularly simple:

$$l_j(x) = \overline{x_j} \ , \quad r_j(x) = x_j \ .$$

$$\left(\bigotimes_{j=0}^{n-1}\left((\overline{x_j}\quad x_j)\cdot Q_i^{-1}\right)\right)\cdot\tilde{f}=\left((\overline{x_0}\quad x_0)\cdot Q_0^{-1}\right)\cdot\begin{pmatrix}\left((\overline{x_1}\quad x_1)\cdot Q_1^{-1}\right)\cdot\begin{pmatrix}\left(\bigotimes_{j=2}^{n-1}\left((\overline{x_j}\quad x_j)\cdot Q_j^{-1}\right)\right)\cdot\tilde{f}|_{00}\\ \left(\bigotimes_{j=2}^{n-1}\left((\overline{x_j}\quad x_j)\cdot Q_j^{-1}\right)\right)\cdot\tilde{f}|_{01}\end{pmatrix}\\ \left((\overline{x_1}\quad x_1)\cdot Q_1^{-1}\right)\cdot\begin{pmatrix}\left(\bigotimes_{j=2}^{n-1}\left((\overline{x_j}\quad x_j)\cdot Q_j^{-1}\right)\right)\cdot\tilde{f}|_{10}\\ \left(\bigotimes_{j=2}^{n-1}\left((\overline{x_j}\quad x_j)\cdot Q_j^{-1}\right)\right)\cdot\tilde{f}|_{11}\end{pmatrix}\end{pmatrix}$$

Figure 2: Unfolding the recursive definition of a Q–transform

Thus, at every node in such a tree, one of the two branches starting at that node is labeled 0 and the other 1. So, these trees actually are *decision* trees; this is not the case for most other transforms.

In the case of the Shannon-transformation, a decomposition of the function F along these lines is known as the *Shannon-expansion* w.r.t. the variable x_j:

$$F=(\overline{x_j}\quad x_j)\,(F|_{x_j=0}\quad F|_{x_j=1})^T$$

For arbitrary Q–transformations we define the Q–expansion with respect to x_j:

$$F=\left((\overline{x_j}\quad x_j)\,Q_j^{-1}\right)\left(Q_j\,(F|_{x_j=0}\quad F|_{x_j=1})^T\right)\ .$$

These expansions just describe the node-operations on the Q–transform trees. Thus, each level of a Q–transform tree can be regarded as a kind of Q_j–expansion with respect to the variable x_j.

The space required to represent boolean functions by Q–transform trees can be greatly reduced in certain situations. Obviously, space can be saved by converting these trees to *directed acyclic graphs*, where identical subtrees are shared. Additional reduction in space arises from the *elimination of unnecessary nodes*. A node in a tree is unnecessary, if both its subtrees are identical. Such a node can be eliminated, after an adjustment is made to the label of the branch preceding this node. This situation is depicted in figure 4. Algebraically, the following term corresponds to the original (left) tree:

$$(l_j(x_j)\quad r_j(x_j))\cdot\begin{pmatrix}(l_{j+1}(x_{j+1})\quad r_{j+1}(x_{j+1}))\cdot\begin{pmatrix}B\\B'\end{pmatrix}\\ (l_{j+1}(x_{j+1})\quad r_{j+1}(x_{j+1}))\cdot\begin{pmatrix}A\\A\end{pmatrix}\end{pmatrix}$$

Here, A, B and B' are the subtrees, where A occurs twice as a subtree of the same node. This expression can be simplified to the following one (for details cf. [11]):

$$\left(l_j(x_j)\quad r_j(x_j)(l_{j+1}(x_{j+1})+r_{j+1}(x_{j+1}))\right)\cdot\begin{pmatrix}(l_{j+1}(x_{j+1})\quad r_{j+1}(x_{j+1}))\cdot\begin{pmatrix}B\\B'\end{pmatrix}\\ A\end{pmatrix}$$

which formalises the reduced (right) tree in fig.4.

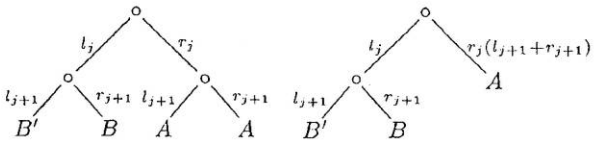

Figure 4: Elimination of unnecessary nodes

The *ordering of the variables* for Q–transform trees has not been considered so far. The variable ordering determines which reductions can be made, and can have a dramatic effect on the size of the final directed acyclic graph. Since this topic has been discussed extensively in the literature [14, 17], we will not discuss it further in this paper.

Now, for any hybrid Q–transformation, we define the corresponding *hybrid Q–transform diagram* to be the Q–transform tree together with the additional operations of node elimination, sharing of common subdiagrams and variable ordering. Various transform diagrams, that appear in the literature, can be classified using this terminology. For example, MTBDDs can be understood as homogeneous Shannon–transform diagrams. Some of the most common diagrams are listed in table 1.

6 Directions for future research

In this paper we provided a uniform algebraic framework for computing spectral transforms [10] and hybrid transform-diagrams (an efficient data-structure for D–valued boolean functions) [7, 8] in an efficient manner. The use of such diagrams has resulted in the development of new verification methods for computer arithmetic [9]. It is clear that these ideas are not confined to applications in digital circuit design. D–valued boolean functions, expressed in terms of hybrid transform diagrams can be used to represent large matrices [6]. Thus, applications of such methods in numerical analysis and linear algebra are obvious. Direct and iterative methods for solving systems of linear equations are typical examples[2, 6]. Many problems in graph theory can also be formulated in terms of matrices and then solved by the methods presented in [2, 6]. Finally, Markov analysis

Matrix	$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	$\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$	$\begin{pmatrix}0 & 1\\ 1 & -1\end{pmatrix}$	$\begin{pmatrix}1 & 0\\ -1 & 1\end{pmatrix}$	$\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$
Name	Shannon	Reed-Muller Positive Davio	Negative Davio	Arithmetic	Walsh
Expansion: $(l(x) \quad r(x))\,v$	$(\overline{x} \quad x)\,v$	$(\overline{x}-x \quad x)\,v$	$(1 \quad \overline{x})\,v$	$(1 \quad x)\,v$	$\frac{1}{2}(1 \quad \overline{x}-x)v$
Homogeneous Diagrams	BDD [1, 4] MTBDD [6]	FDD [5, 16]	—	BMD [5] ACDD [19]	WDD [19]
Heterogeneous Diagrams	←——— KDD [13] ———→				
	←——— Hybrid transform diagrams [7] ———→				

Table 1: Common matrices and transform diagrams

and probabilistic model checking involve huge matrices, and may ultimately benefit from these techniques [2, 3].

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