Assignment 2: Transformation and Viewing

15-462 Graphics I
Spring 2002
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Out January 31
Due February 7 before lecture
50 points

- The work must be all your own.
- The assignment is due **before lecture** on Thursday, February 7.
- Be explicit, define your symbols, and explain your steps.
  This will make it a lot easier for us to assign partial credit.
- Use geometric intuition together with trigonometry and linear algebra.
- Verify whether your answer is meaningful with a simple example.

1 Three-Dimensional Homogeneous Coordinates (15 pts)

If we are interested only in two-dimensional graphics, we can use three-dimensional homogeneous coordinates by representing a point **P** by $[x\ y\ 1]^T$ and a vector **v** by $[\alpha\ \beta\ 0]^T$.

1. Find the matrix representation of a counter-clockwise rotation by θ degrees about the origin.
2. Find the translation matrix for given displacement vector $[\delta_x\ \delta_y\ 0]^T$
3. Find the scaling matrix for factors α_x and α_y.
4. Find the x-shear matrix for shear angle θ.
5. Derive the explicit transformation matrix for a rotation about the axis specified by a point $\mathbf{p_0} = [x_0\ y_0\ 1]^T$ and a unit vector $\mathbf{u} = [\alpha_x\ \alpha_y\ 0]^T$.
6. Show how the x-shear matrix can be represented as a composition of rotations, scalings, and translations.

2 Rigid Body Transformations (20 pts)

A *rigid body transformation* may rotate and move, but not reflect, re-scale, or otherwise distort an object. We first investigate these in two dimensions (see Problem 1) and then generalize to three dimensions. Your tests should avoid trigonometric functions or their inverses.

1. Devise a test whether a given 3×3 transformation matrix in homogeneous coordinates is a rigid body transformation in 2 dimensions.
2. Generalize your test to check if a given 4×4 transformation matrix in homogeneous coordinates is a rigid body transformation in 3 dimensions.

3 Viewing Transformations (15 pts)

Assume the function `void earth ();` draws a three dimensional model of the earth with the south pole at the origin, the north pole at the point $(0, 1, 0)$, and the Greenwich meridian (0^o longitude) pointing in the z-direction. We are interested in drawing the earth as seen from a point in space with a given *longitude* and *latitude* (specified in degrees) and given *distance* from the surface of the earth. We want to be looking down into the direction of the earth's center and have a square viewport that should cover a field of vision of 30^o degrees. We are assuming the earth is a perfect sphere.

1. Does the specification above uniquely determine the perspective viewing transformation? Explain if there are additional degrees of freedom.
2. Give code for a function

   ```
   void viewEarth (float longitude, float latitude, float distance);
   ```

   and carefully explain the reasoning behind your solution. If there are additional degrees of freedom, set them to some reasonable values. Your function should call `earth ();` to draw the earth.