15-462 Computer Graphics I

Midterm Examination

March 5, 2002

Name: ______________________________

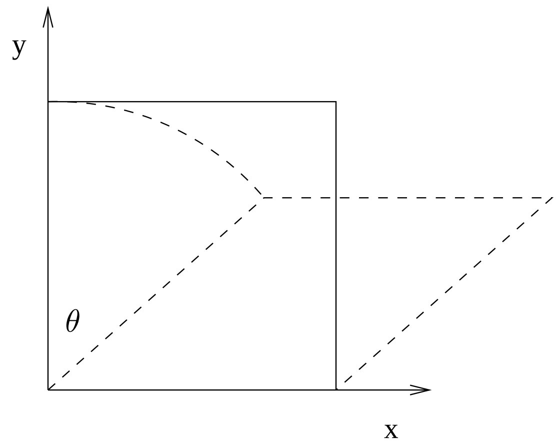
Andrew User ID: ____________________

- This is a closed-book exam; only one double-sided sheet of notes is permitted.
- Write your answer legibly in the space provided.
- There are 10 pages in this exam, including 3 worksheets.
- It consists of 3 questions worth a total of 100 points.
- You have 80 minutes for this exam.

Problem 1	Problem 2	Problem 3	Total
40	30	30	100

1. Linear Transformations (40 pts)

Consider the following *skewing transformation*.



1. (20 pts) Show the 2-dimensional skewing transformation matrix for a given angle θ in homogeneous coordinates. This should be a 3×3 matrix. Explain your reasoning.

2. (20 pts) Show how the skewing transformation can be represented as the composition of a scaling and a shearing transformation. Write out the auxiliary transformations explicitly as matrices and verify that the composition yields the skewing matrix from part 1.

2. Projections (30 pts)

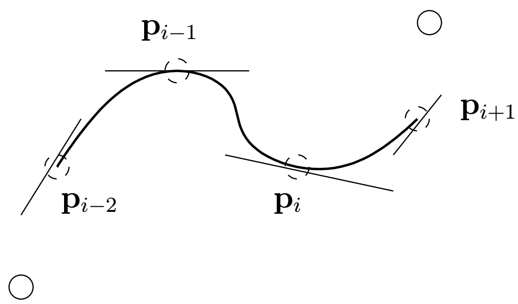
In the textbook, the perspective projection matrix is given for the center of projection at the origin and the projection plane at $z = -d$ for a given distance d. In this problem we will develop a different perspective projection matrix the clarifies the relation between orthogonal and perspective projections. Your answers should be 4×4 matrices in homogeneous coordinates.

1. (20 pts) Give the perspective projection matrix with the center of projection at $x = 0, y = 0, z = d$ and the projection plane $z = 0$. Draw a picture to aid your reasoning.

2. (10 pts) Give the orthogonal projection matrix onto the plane $z = 0$ and verify that we obtain the orthogonal projection matrix as the limit of the perspective projection matrix as d goes to infinity.

3. Splines (30 points)

In this problem with explore Catmull-Rom splines. In two dimension, they are guaranteed to interpolate the interior m points, given $m + 2$ control points. Besides interpolation, we require that the tangent vector at each interior control point $\mathbf{p}_k$ is the average of the vectors from $\mathbf{p}_{k-1}$ to $\mathbf{p}_k$ and from $\mathbf{p}_k$ to $\mathbf{p}_{k+1}$.



1. (20 pts) Set up 4 equations that determine the Catmull-Rom geometry matrix, assuming we are trying to draw the segment from $\mathbf{p}_{i-1}$ to $\mathbf{p}_i$. For each, briefly note the geometric origin of the equation. You do not have to solve your equations.

2. (5 pts) Explain to what extent Catmull-Rom splines allow local control.

3. (5 pts) Do Catmull-Rom splines have the convex hull property?

Worksheet

Worksheet

Worksheet