

15-462 Computer Graphics I
Lecture 18

Radiosity

Measures of Illumination
The Radiosity Equation
Form Factors
Radiosity Algorithms
Handout: [Watt & Watt, Ch 11]

April 9, 2002
Frank Pfenning
Carnegie Mellon University

<http://www.cs.cmu.edu/~fp/courses/graphics/>

Local vs. Global Illumination

- Local illumination: Phong model (OpenGL)
 - Light to surface to viewer
 - No shadows, interreflections
 - Fast enough for interactive graphics
- Global illumination: Ray tracing
 - Multiple specular reflections and transmissions
 - Only one step of diffuse reflection
- Global illumination: Radiosity
 - All diffuse interreflections; shadows
 - Advanced: combine with specular reflection

Image vs. Object Space

- Image space: Ray tracing
 - Trace backwards from viewer
 - View-dependent calculation
 - Result: rasterized image (pixel by pixel)
- Object space: Radiosity
 - Assume only diffuse-diffuse interactions
 - View-independent calculation
 - Result: 3D model, color for each surface patch
 - Can render with OpenGL

04/09/2002

15-462 Graphics I

3

Classical Radiosity Method

- Divide surfaces into patches (elements)
- Model light transfer between patches as system of linear equations
- Important assumptions:
 - Reflection and emission are diffuse
 - Recall: diffuse reflection is equal in all directions
 - So radiance is independent of direction
 - No participating media (no fog)
 - No transmission (only opaque surfaces)
 - Radiosity is constant across each element
 - Solve for R, G, B separately

04/09/2002

15-462 Graphics I

4

Outline

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

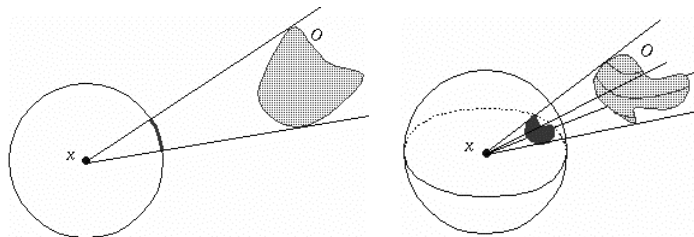
04/09/2002

15-462 Graphics I

5

Solid Angle

- 2D angle subtended by object O from point x :
 - Length of projection onto unit circle at x
 - Measured in radians (0 to 2π)
- 3D solid angle subtended by O from point x :
 - Area of of projection onto unit sphere at x
 - Measured in steradians (0 to 4π)



J. Stewart

04/09/2002

15-462 Graphics I

6

Radiant Power and Radiosity

- Radiant power P
 - Rate at which light energy is transmitted
 - Dimension: power = energy / time
- Flux density Φ
 - Radiant power per unit area of the surface
 - Dimension: power / area
- Irradiance E : incident flux density of surface
- Radiosity B : exitant flux density of surface
 - Dimension: power / area
- Flux density at a point $\Phi(x) = dP/dx$

04/09/2002

15-462 Graphics I

7

Power at Point in a Direction

- Radiant intensity I
 - Power radiated per unit solid angle by point source
 - Dimension: power / solid angle
- Radiant intensity in direction ω
 - $I(\omega) = dP/d\omega$
- Radiance $L(x, \omega)$
 - Flux density at point x in direction ω
 - Dimension: power / (area \times solid angle)

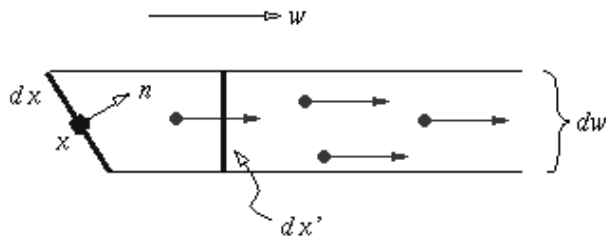
04/09/2002

15-462 Graphics I

8

Radiance

- Measured across surface in direction ω



J. Stewart '98

- For angle θ between ω and normal \mathbf{n}

$$L(x, \omega) = \frac{d^2P}{d\omega dx'} = \frac{d^2P}{d\omega \cos \theta dx}$$

04/09/2002

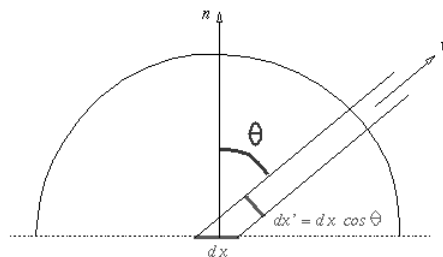
15-462 Graphics I

9

Radiosity and Radiance

- Radiosity $B(x) = dP / dx$
- Radiance $L(x, \omega) = d^2P / d\omega \cos \theta dx$
- Let Ω be set of all directions above x

$$B(x) = \int_{\Omega} L(x, \omega) \cos \theta d\omega$$



04/09/2002

15-462 Graphics I

10

Outline

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

04/09/2002

15-462 Graphics I

11

Balance of Energy

- Lambertian surfaces (ideal diffuse reflector)
- Divided into n elements
- Variables
 - A_i Area of element i (computable)
 - B_i Radiosity of element i (unknown)
 - E_i Radiant emitted flux density of element i (given)
 - ρ_i Reflectance of element i (given)
 - F_{ji} Form factor from j to i (computable)

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^n F_{ji} A_j B_j$$

04/09/2002

15-462 Graphics I

12

Form Factors

- Form factor F_{ij} : Fraction of light leaving element i arriving at element j
- Depends on
 - Shape of patches i and j
 - Relative orientation of both patches
 - Distance between patches
 - Occlusion by other patches

04/09/2002

15-462 Graphics I

13

Form Factor Equation

- Polar angles θ and θ' between normals and ray between x and y
- Visibility function $v(x,y) = 0$ if ray from x to y is occluded, $v(x,y) = 1$ otherwise
- Distance r between x and y

$$A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos \theta \cos \theta'}{\pi r^2} v(x, y) dy dx$$

04/09/2002

15-462 Graphics I

14

Reciprocity

- Symmetry of form factor

$$A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos \theta \cos \theta'}{\pi r^2} v(x, y) dy dx = A_j F_{ji}$$

- Divide earlier radiosity equation

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^n F_{ji} A_j B_j$$

$$\begin{aligned} \text{by } A_i \quad B_i &= E_i + \rho_i \sum_j (F_{ji} A_j / A_i) B_j \\ &= E_i + \rho_i \sum_j F_{ij} B_j \end{aligned}$$

04/09/2002

15-462 Graphics I

15

Radiosity as a Linear System

- Restate radiosity equation $B_i - \rho_i \sum_j F_{ij} B_j = E_i$
- In matrix form

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & \rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & \rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & \rho_n F_{n2} & \cdots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

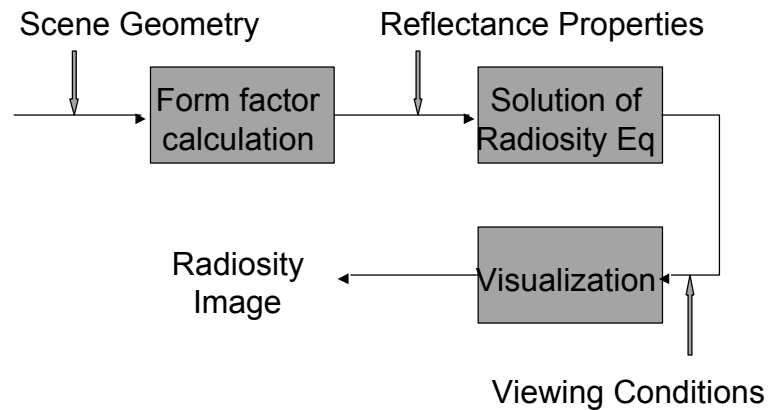
- Known: reflectances ρ_i , form factors F_{ij} , emissions E_i
- Unknown: Radiosities B_i
- n linear equations in n unknowns

04/09/2002

15-462 Graphics I

16

Radiosity "Pipeline"



04/09/2002

15-462 Graphics I

17

Visualization

- Radiosity solution is viewer independent
- Can exploit graphics hardware to obtain image
- Convert color on patch to vertex color
- Easy part of radiosity method

04/09/2002

15-462 Graphics I

18

Outline

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

Computing Form Factors

- Visibility critical
- Two principal methods
 - Hemicube: exploit z-buffer hardware
 - Ray casting (can be slow)
 - Both exhibit aliasing effects
- For inter-visible elements
 - Many special cases can be solved analytically
 - Avoid full numeric approximation of double integral

Hemicube Algorithm

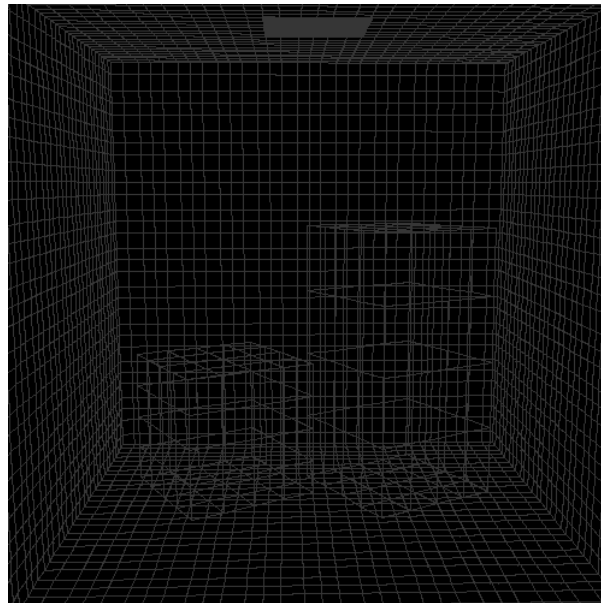
- Render model onto a hemicube as soon from the center of a patch
- Store patch identifiers j instead of color
- Use z-buffer to resolve visibility
- Efficiently implementable in hardware
- Examples of antialiasing [Chandran et al.]

04/09/2002

15-462 Graphics I

21

Wireframe

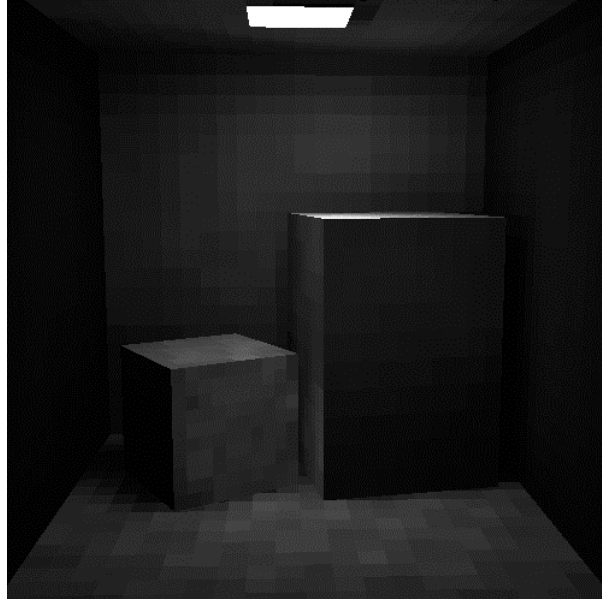


04/09/2002

15-462 Graphics I

22

Classical, No Intensity Interpolation

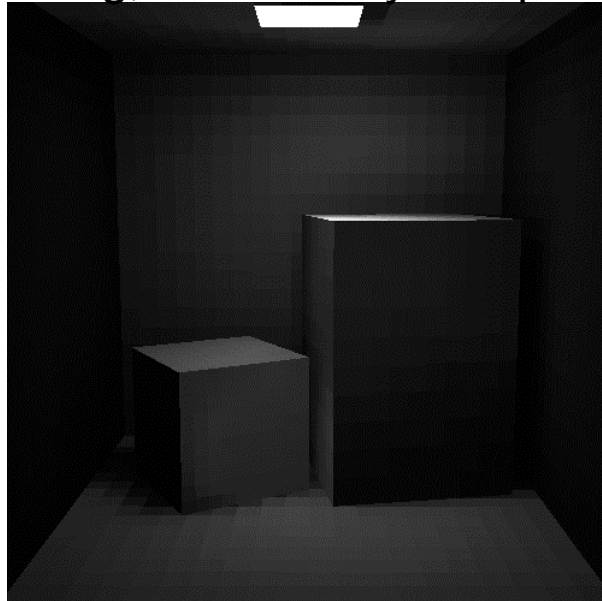


04/09/2002

15-462 Graphics I

23

Antialiasing, No Intensity Interpolation

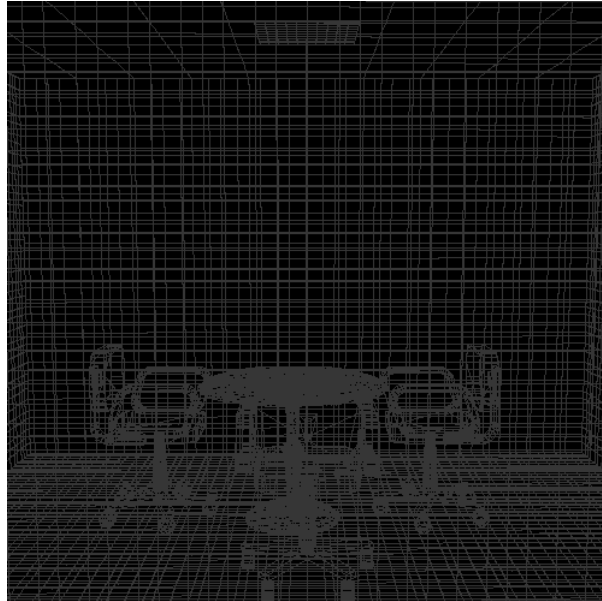


04/09/2002

15-462 Graphics I

24

Wireframe



04/09/2002

15-462 Graphics I

25

Classical, Resolution 300

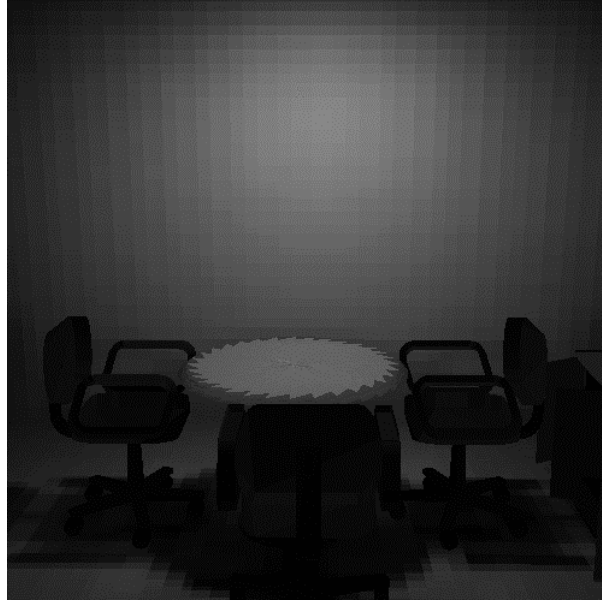


04/09/2002

15-462 Graphics I

26

Classical, Resolution 1200



04/09/2002

15-462 Graphics I

27

Classical, Resolution 2500



04/09/2002

15-462 Graphics I

28

Supersampling, Resolution 100

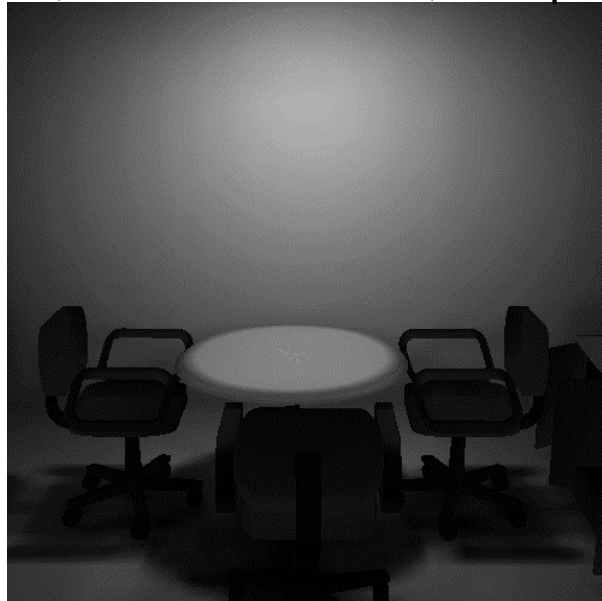


04/09/2002

15-462 Graphics I

29

Classical, Resolution 2500, Interpolated

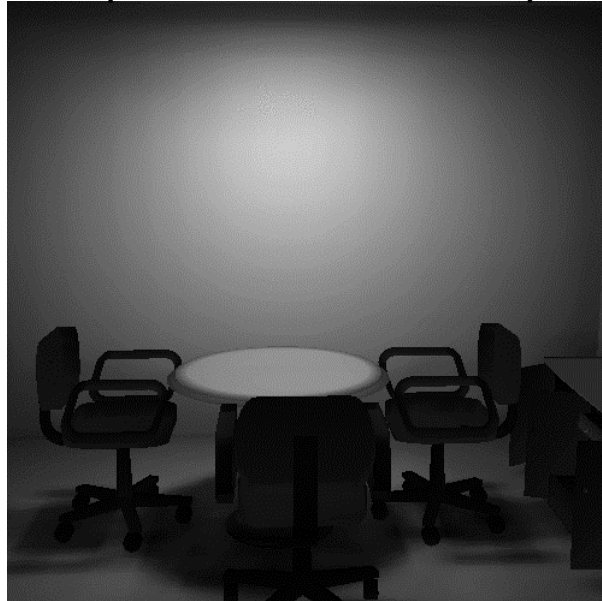


04/09/2002

15-462 Graphics I

30

Supersampled, Res 100, Interpolated



04/09/2002

15-462 Graphics I

31

Outline

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

04/09/2002

15-462 Graphics I

32

Radiosity Equation Revisited

- Direct form

$$B_i = E_i + \rho_i \sum_j F_{ij} B_j$$

- As matrix equation

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & \rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & \rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & \rho_n F_{n2} & \cdots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

- Unknown: radiosity B_i
- Known: emission E_i , form factor F_{ij} , reflect. ρ_i

04/09/2002

15-462 Graphics I

33

Classical Radiosity Algorithms

- Matrix Radiosity
 - Diagonally dominant matrix
 - Use Gauss-Seidel iterative solution
 - Time and space complexity is $O(n^2)$ for n elements
 - Memory cost excessive
- Progressive Refinement Radiosity
 - Solve equations incrementally with form factors
 - Time complexity is $O(n \cdot s)$ for s iterations
 - Used more commonly (space complexity $O(n)$)

04/09/2002

15-462 Graphics I

34

Matrix Radiosity

- Compute all form factors F_{ij}
- Make initial approximation to radiosity
 - Emitting elements $B_i = E_i$
 - Other elements $B_i = 0$
- Apply equation to get next approximation
$$B'_i = E_i + \rho_i \sum_j F_{ij} B_j$$
- Iterate with new approximation
- Intuitively
 - Gather incoming light for each element i
 - Base new estimate on previous estimate

04/09/2002

15-462 Graphics I

35

Radiosity Summary

- Assumptions
 - Opaque Lambertian surfaces (ideal diffuse)
 - Radiosity constant across each element
- Radiosity computation structure
 - Break scene into patches
 - Compute form factors between patches
 - Lighting independent
 - Solve linear radiosity equation
 - Viewer independent
 - Render using standard hardware

04/09/2002

15-462 Graphics I

36

Lecture Summary

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

04/09/2002

15-462 Graphics I

37

Preview

- Next Lecture
 - Radiosity refinements
 - Combining ray tracing and radiosity
- Assignment 7 (Ray Tracer) due April 23
- Different from OpenGL programming (150 pts)

04/09/2002

15-462 Graphics I

38