

15-462 Computer Graphics I

Lecture 21

Visualization

Height Fields and Contours
Scalar Fields
Volume Rendering
Vector Fields

[Angel Ch. 12]

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Scientific Visualization

- Generally do not start with a 3D model
- Must deal with very large data sets
 - MRI, e.g. $512 \times 512 \times 200 \approx 50\text{MB}$ points
 - Visible Human $512 \times 512 \times 1734 \approx 433\text{ MB}$ points
- Visualize both real-world and simulation data
- User interaction
- Automatic search

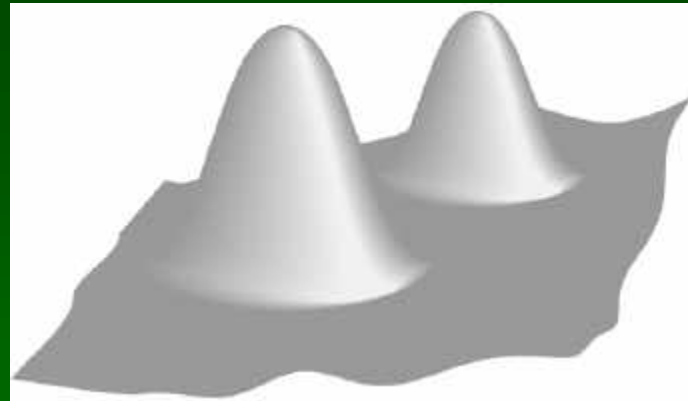
Types of Data

- Scalar fields (3D volume of scalars)
 - E.g., x-ray densities (MRI, CT scan)
- Vector fields (3D volume of vectors)
 - E.g., velocities in a wind tunnel
- Tensor fields (3D volume of tensors [matrices])
 - E.g., stresses in a mechanical part [[Angel 12.7](#)]
- Static or through time

Height Field

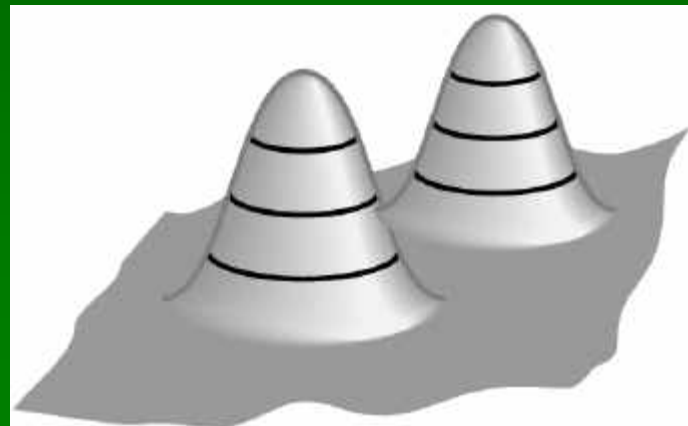
- Visualizing an explicit function

$$z = f(x,y)$$



- Adding contour curves

$$g(x,y) = c$$



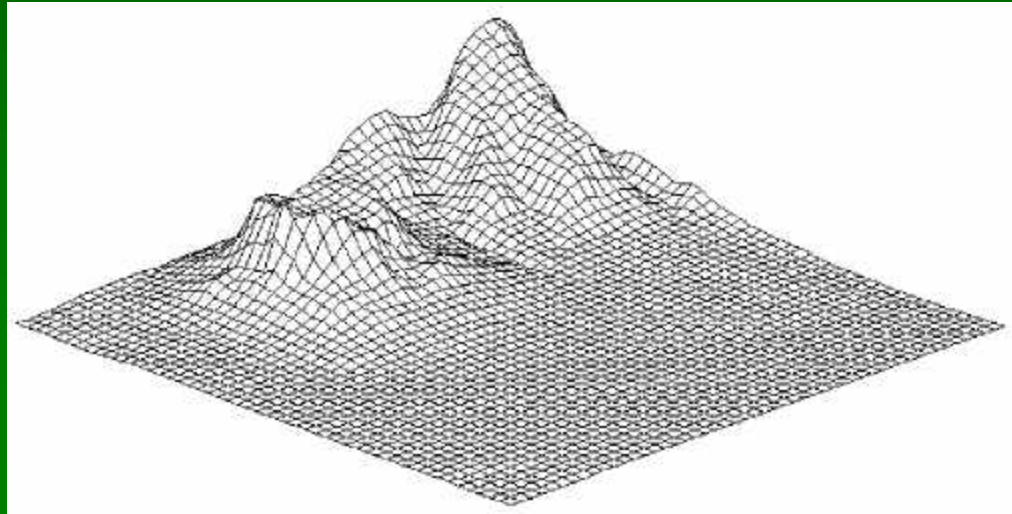
Meshes

- Function is sampled (given) at $x_i, y_j, 0 \leq i, j \leq n$
- Assume equally spaced

$$F(u, v) = c(u)c(v) \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} f(x, y) \cos \frac{(2x+1)u\pi}{2n} \cos \frac{(2y+1)v\pi}{2n}$$

where $c(u) = 1/\sqrt{n}$ if $u = 0$, $c(u) = \sqrt{2/n}$ otherwise

- Generate quadrilateral or triangular mesh
- [Asst 1]



Contour Curves

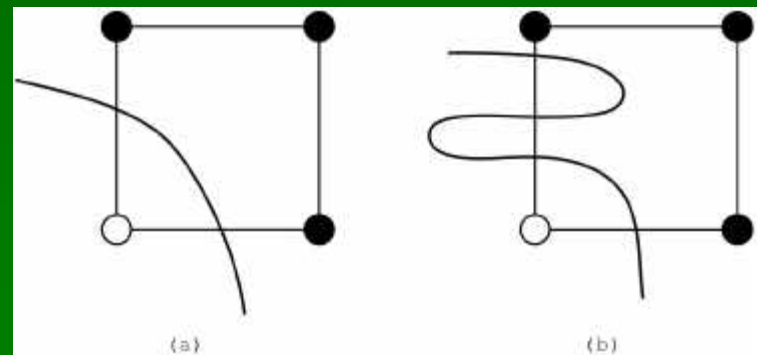
- Recall: implicit curve $f(x,y) = 0$
- $f(x,y) < 0$ inside, $f(x,y) > 0$ outside
- Here: contour curve at $f(x,y) = c$
- Sample at regular intervals for x,y

$$\begin{aligned}x_i &= x_0 + i\Delta x \\y_j &= y_0 + j\Delta y\end{aligned}$$

- How can we draw the curve?

Marching Squares

- Sample function f at every grid point x_i, y_j
- For every point $f_{ij} = f(x_i, y_j)$ either $f_{ij} \leq c$ or $f_{ij} > c$
- Distinguish those cases for each corner x
 - White: $f_{ij} \leq c$
 - Black: $f_{ij} > c$
- Now consider cases for curve
- Assume “smooth”
- Ignore $f_{ij} = 0$

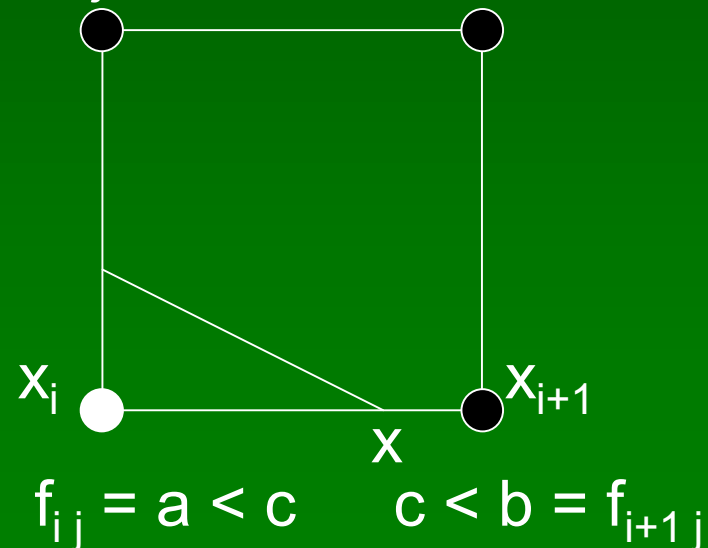


Interpolating Intersections

- Approximate intersection
 - Midpoint between x_i, x_{i+1} and y_j, y_{j+1}
 - Better: interpolate
- If $f_{ij} = a$ is closer to c than $b = f_{i+1j}$ then intersection is closer to (x_i, y_j) :

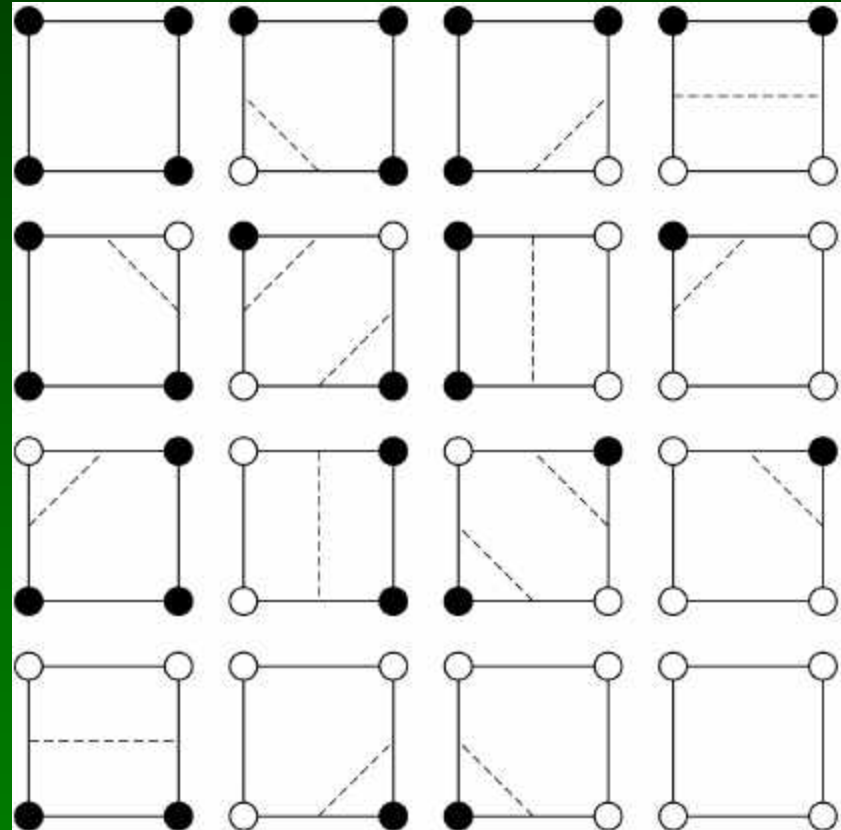
$$\frac{x - x_i}{x_{i+1} - x} = \frac{c - a}{b - c}$$

- Analogous calculation for y direction

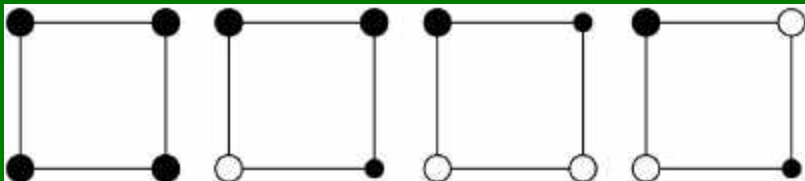


Cases for Vertex Labels

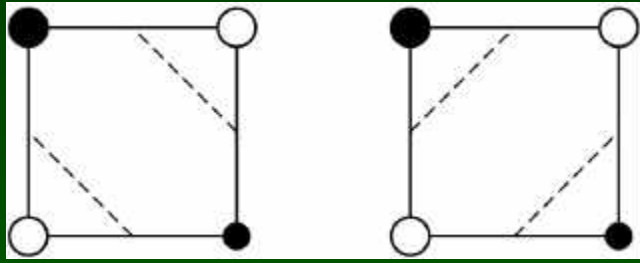
16 cases for vertex labels



4 unique mod. symmetries

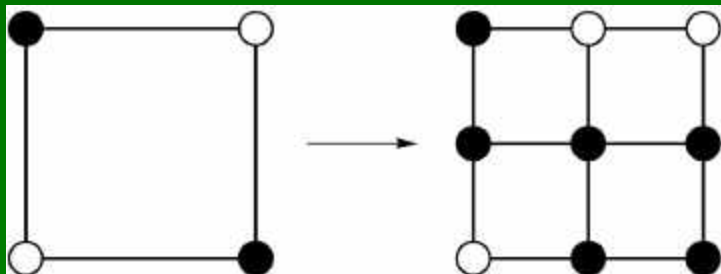
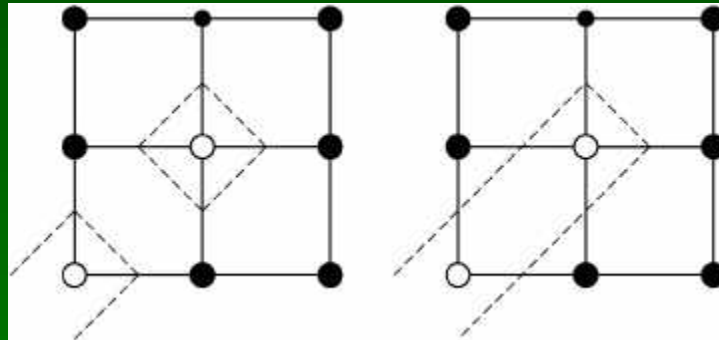


Ambiguities of Labelings



Ambiguous labels

Different resulting contours

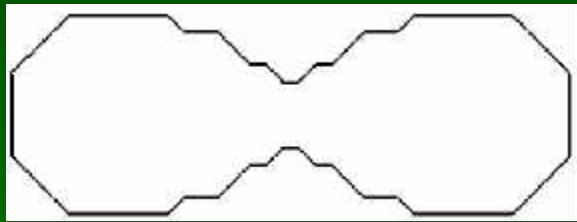


Resolution by subdivision
(where possible)

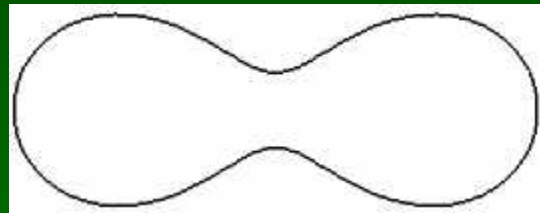
Marching Squares Examples

- Ovals of Cassini, 50×50 grid

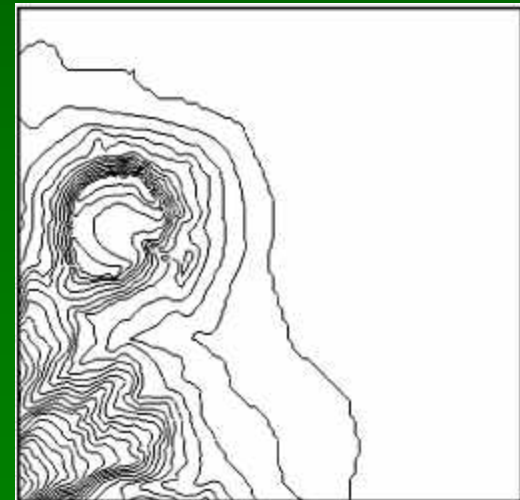
$$f(x, y) = (x^2 + y^2 + a^2)^2 - 4a^2x^2 - b^4$$
$$a = 0.49, b = 0.5$$



Midpoint



Interpolation



Contour plot of Honolulu data

Outline

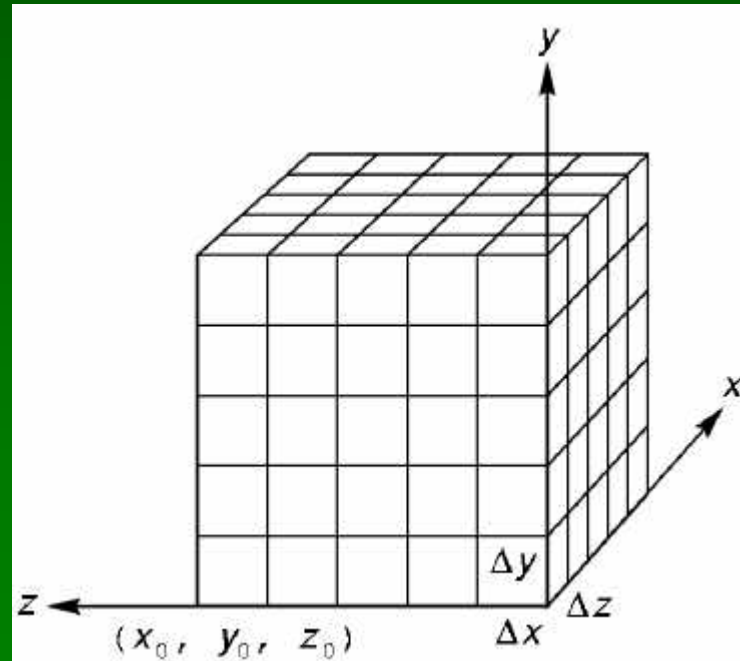
- Height Fields and Contours
- **Scalar Fields**
- Volume Rendering
- Vector Fields

Scalar Fields

- Volumetric data sets
- Example: tissue density
- Assume again regularly sampled

$$\begin{aligned}x_i &= x_0 + i\Delta x \\y_j &= y_0 + j\Delta y \\z_k &= z_0 + k\Delta z\end{aligned}$$

- Represent as **voxels**

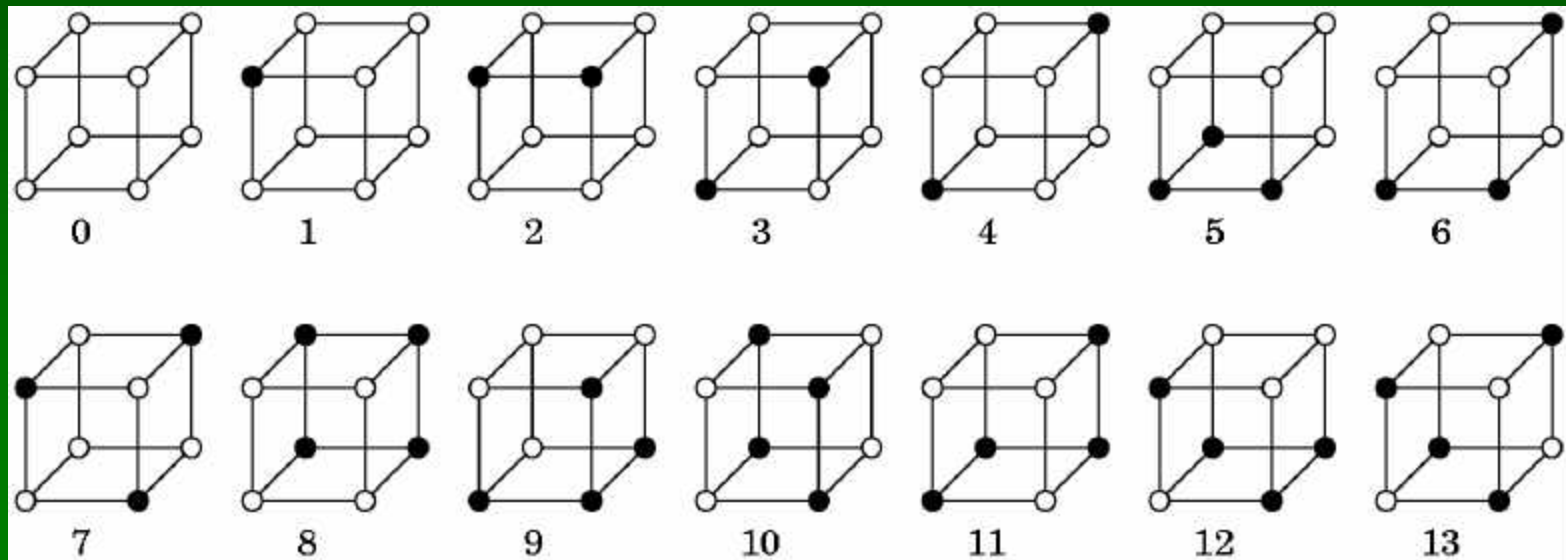


Isosurfaces

- $f(x,y,z)$ represents volumetric data set
- Two rendering methods
 - Isosurface rendering
 - Direct volume rendering (use all values [next])
- **Isosurface** given by $f(x,y,z) = c$
- Recall implicit surface $g(x, y, z)$:
 - $g(x, y, z) < 0$ inside
 - $g(x, y, z) = 0$ surface
 - $g(x, y, z) > 0$ outside
- Generalize right-hand side from 0 to c

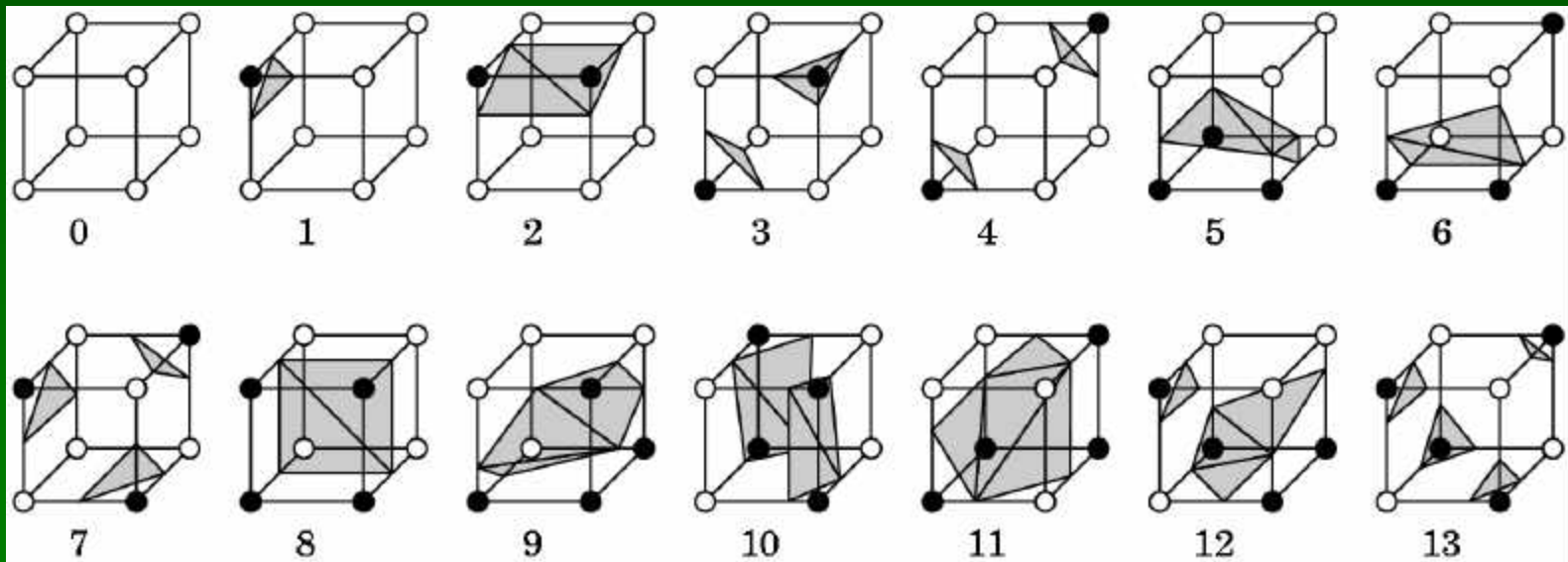
Marching Cubes

- Display technique for isosurfaces
- 3D version of marching squares
- 14 cube labelings (after elimination symmetries)



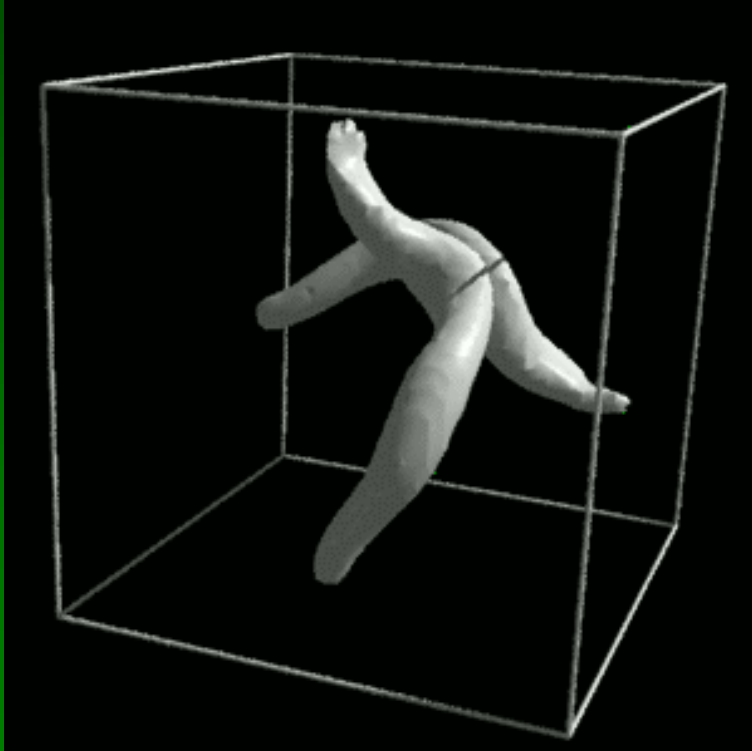
Marching Cube Tessellations

- Generalize marching squares, just more cases
- Interpolate as in 2D
- Ambiguities similar to 2D

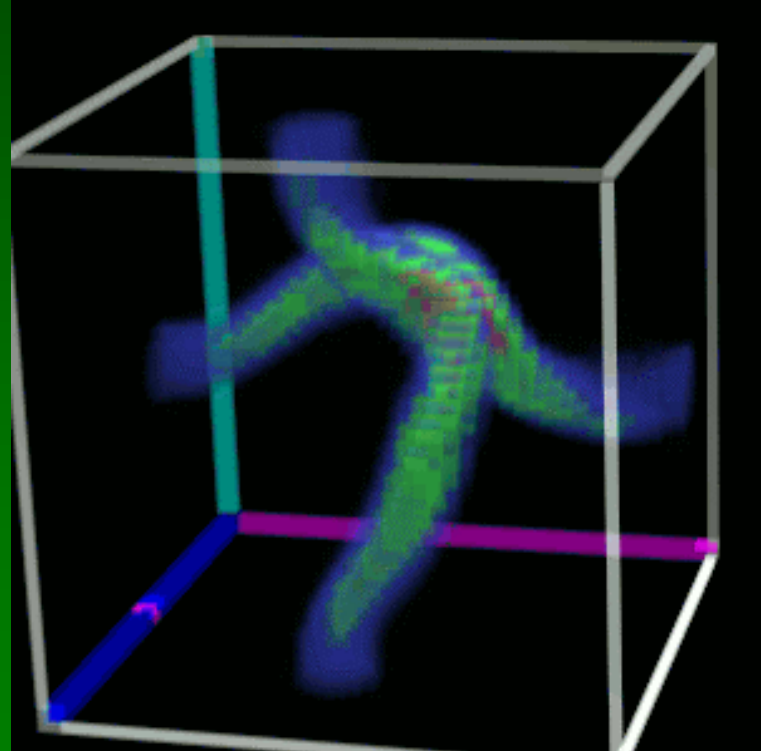


Volume Rendering

- Sometimes isosurfaces are unnatural
- Use all voxels and transparency (α -values)



Ray-traced isosurface



Volume rendering

Surface vs. Volume Rendering

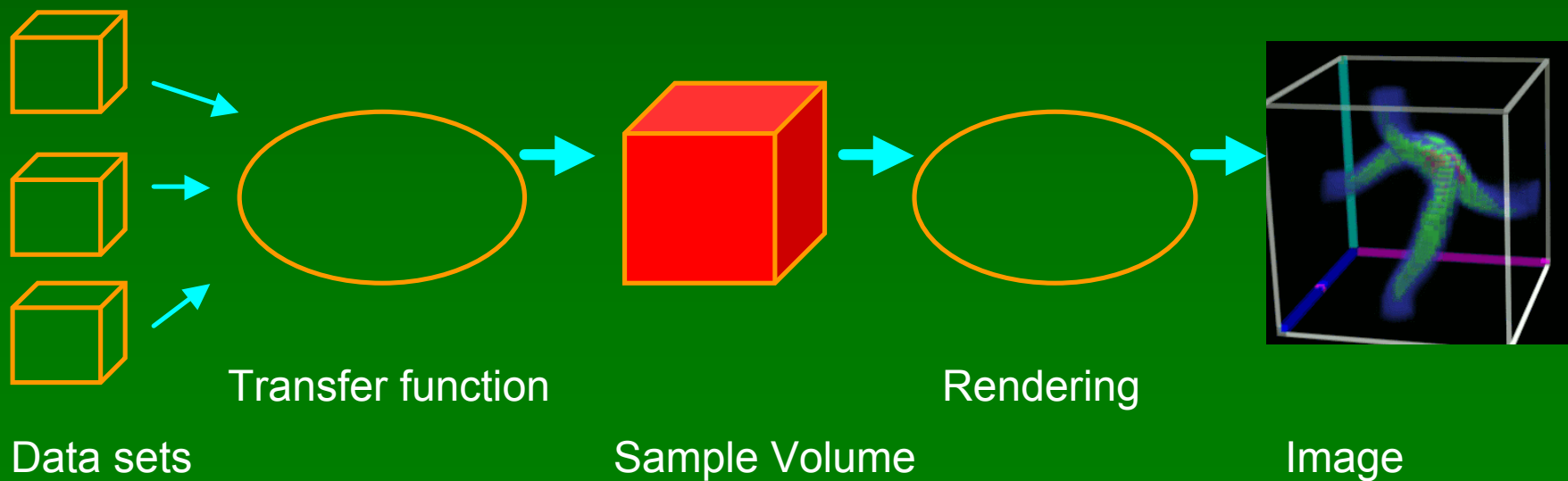
- 3D model of surfaces
- Convert to triangles
- Draw primitives
- Lose or disguise data
- Good for opaque objects
- Scalar field in 3D
- Convert to RGBA values
- Render volume “directly”
- See data as given
- Good for complex objects

Sample Applications

- Medical
 - Computed Tomography (CT)
 - Magnetic Resonance Imaging (MRI)
 - Ultrasound
- Engineering and Science
 - Computational Fluid Dynamic (CFD)
 - Aerodynamic simulations
 - Meteorology
 - Astrophysics

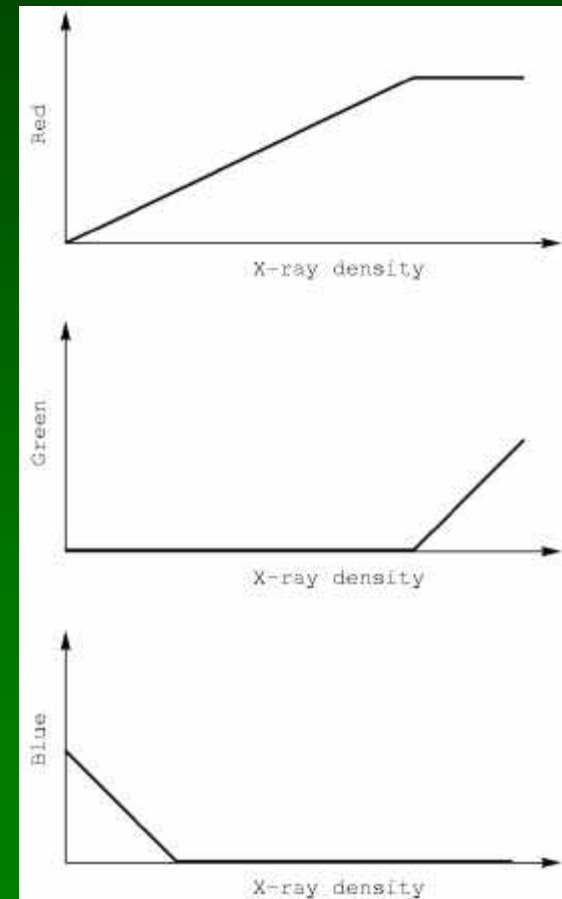
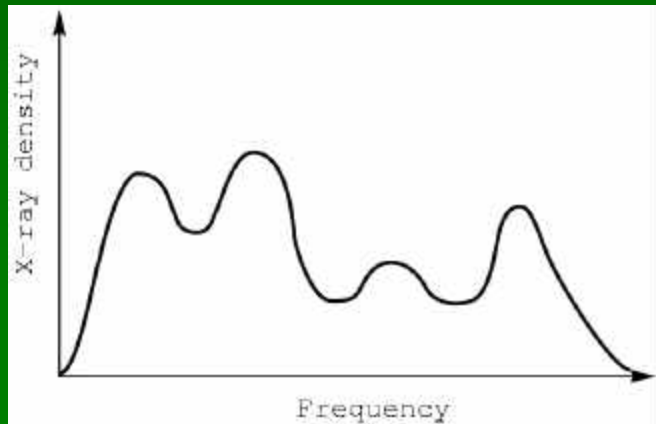
Volume Rendering Pipeline

- Transfer function: from data set to colors and opacities
 - Example: $256 \times 256 \times 64 \times 2 = 4$ MB
 - Example: use colormap (8 bit color, 8 bit opacity)

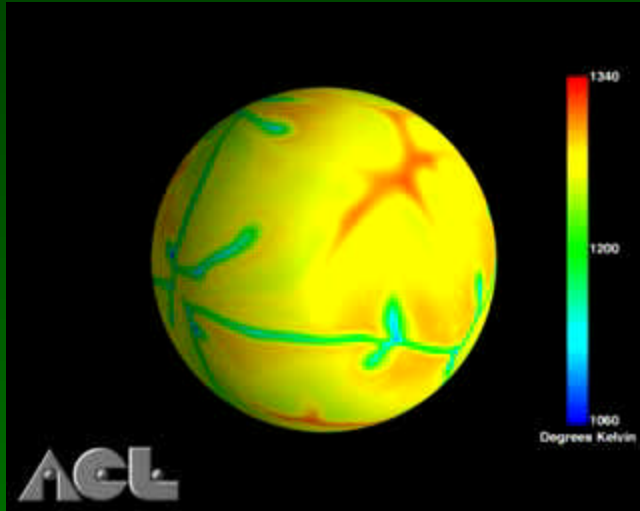


Transfer Functions

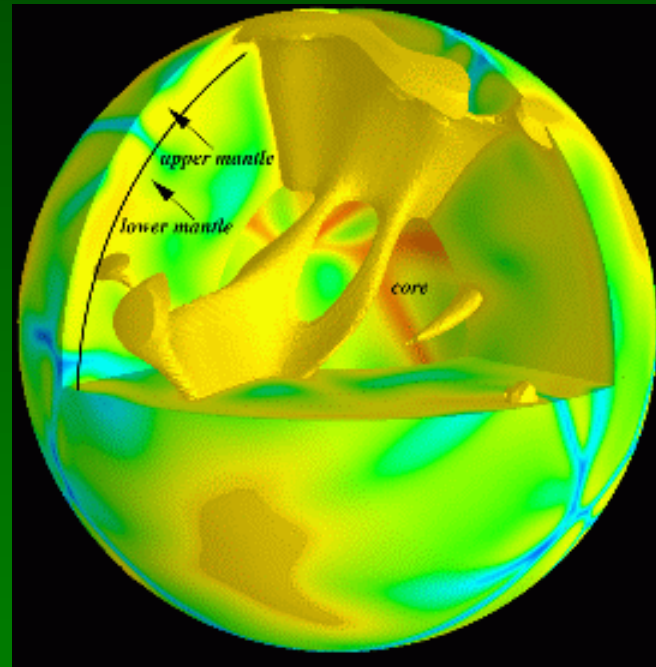
- Transform scalar data values to RGBA values
- Apply to every voxel in volume
- Highly application dependent
- Start from data **histogram**
- Opacity for emphasis



Transfer Function Example

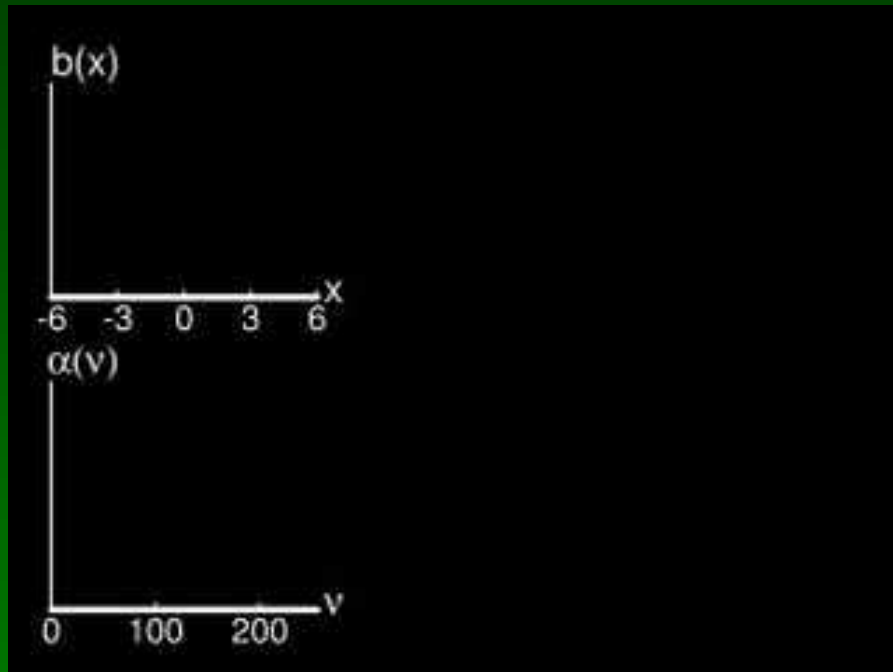


Mantle Convection



Scientific Computing and Imaging (SCI)
University of Utah

Transfer Function Example



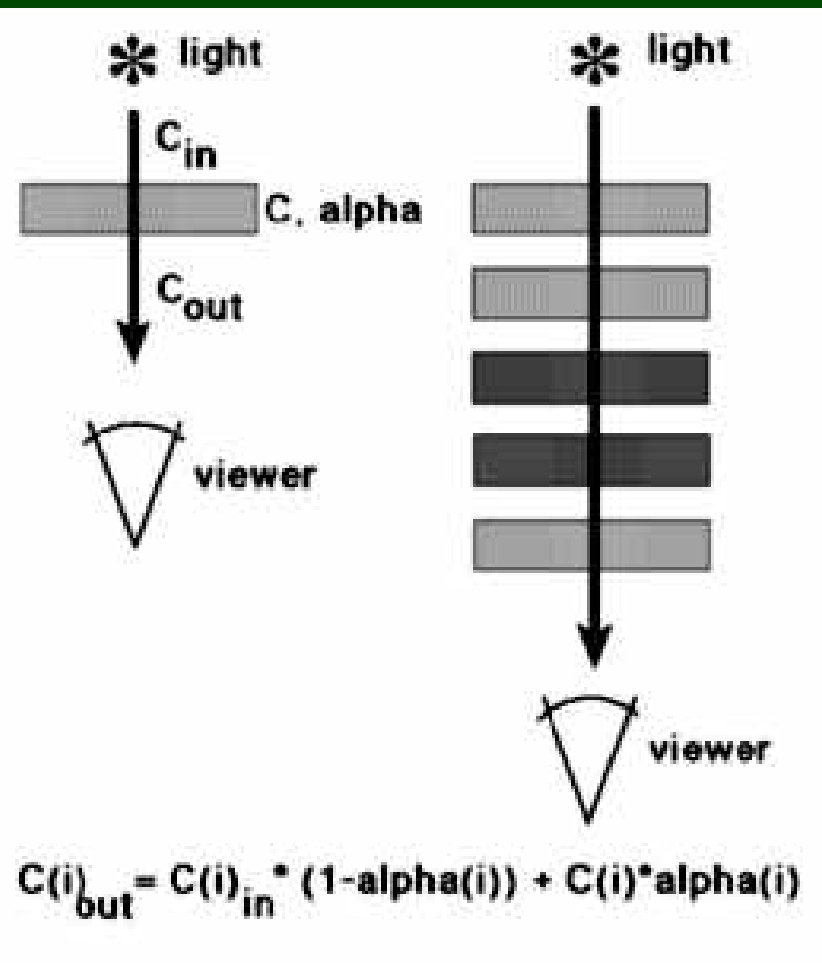
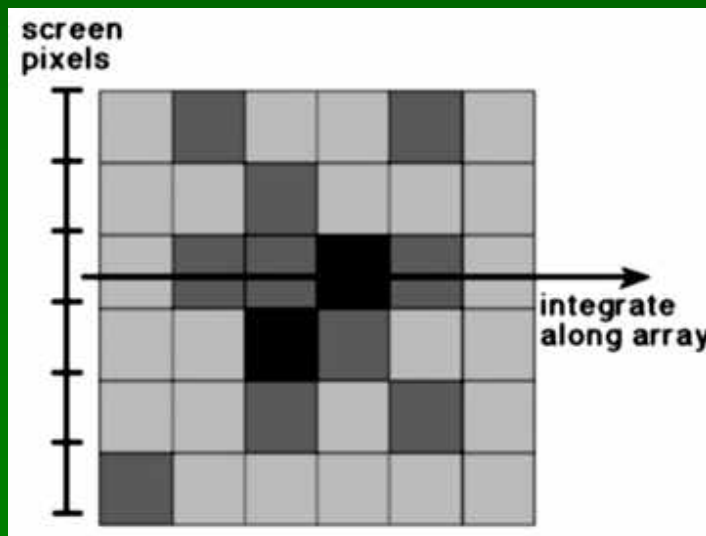
G. Kindlmann

Volume Ray Casting

- Three volume rendering techniques
 - Volume ray casting
 - Splatting
 - 3D texture mapping
- Ray Casting
 - Integrate color through volume
 - Consider lighting (surfaces?)
 - Use regular x,y,z data grid when possible
 - Finite elements when necessary (e.g., ultrasound)
 - 3D-rasterize geometrical primitives

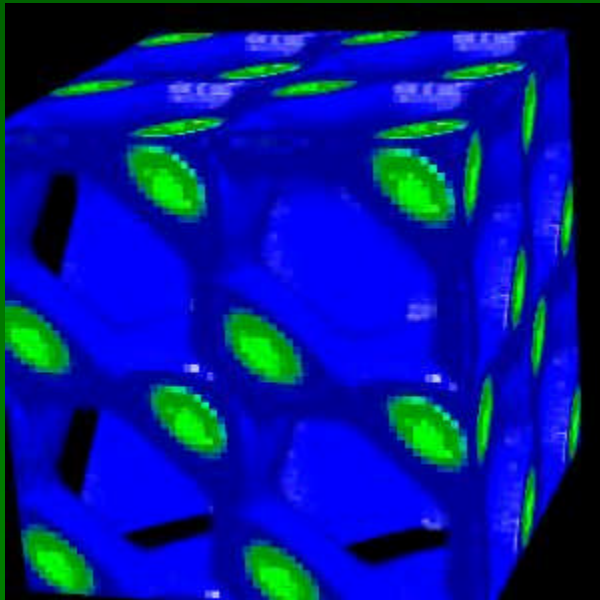
Accumulating Opacity

- $\alpha = 1.0$ is opaque
- Composit multiple layers according to opacity
- Use local gradient of opacity to detect surfaces for lighting

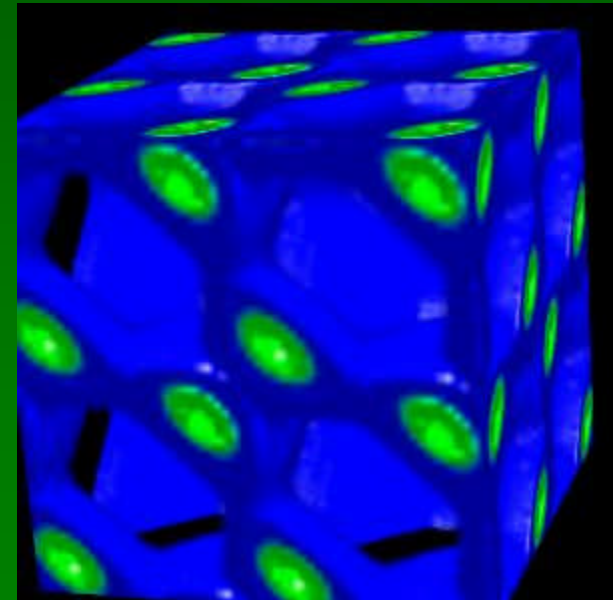


Trilinear Interpolation

- Interpolate to compute RGBA away from grid
- Nearest neighbor yields blocky images
- Use **trilinear interpolation**
- 3D generalization of bilinear interpolation



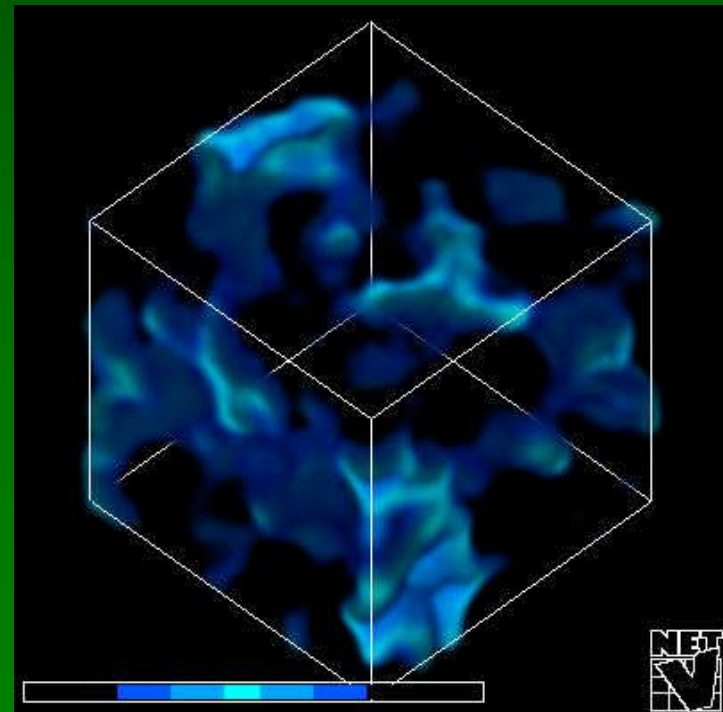
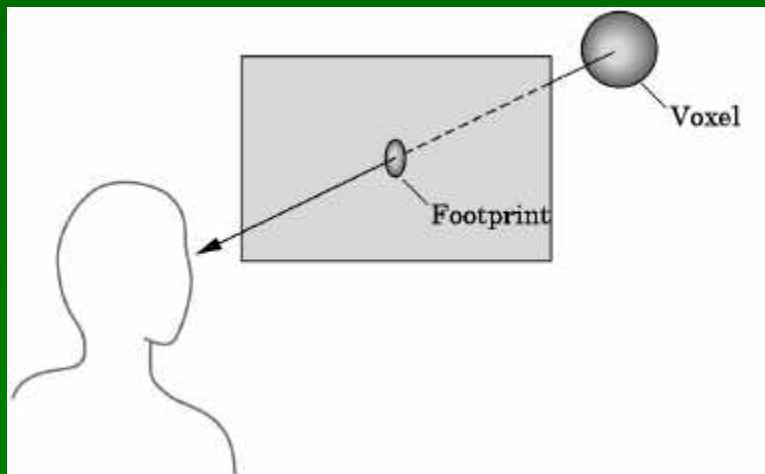
Nearest
neighbor



Trilinear
interpolation

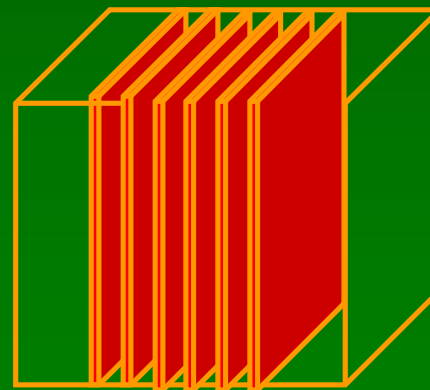
Splatting

- Alternative to ray tracing
- Assign shape to each voxel (e.g., Gaussian)
- Project onto image plane (**splat**)
- Draw voxels back-to-front
- Composite (α -blend)



3D Textures

- Alternative to ray tracing, splatting
- Build a 3D texture (including opacity)
- Draw a stack of polygons, back-to-front
- Efficient if supported in graphics hardware
- Few polygons, much texture memory



3D RGBA texture

Draw back to front

Example: 3D Textures



Emil Praun'01

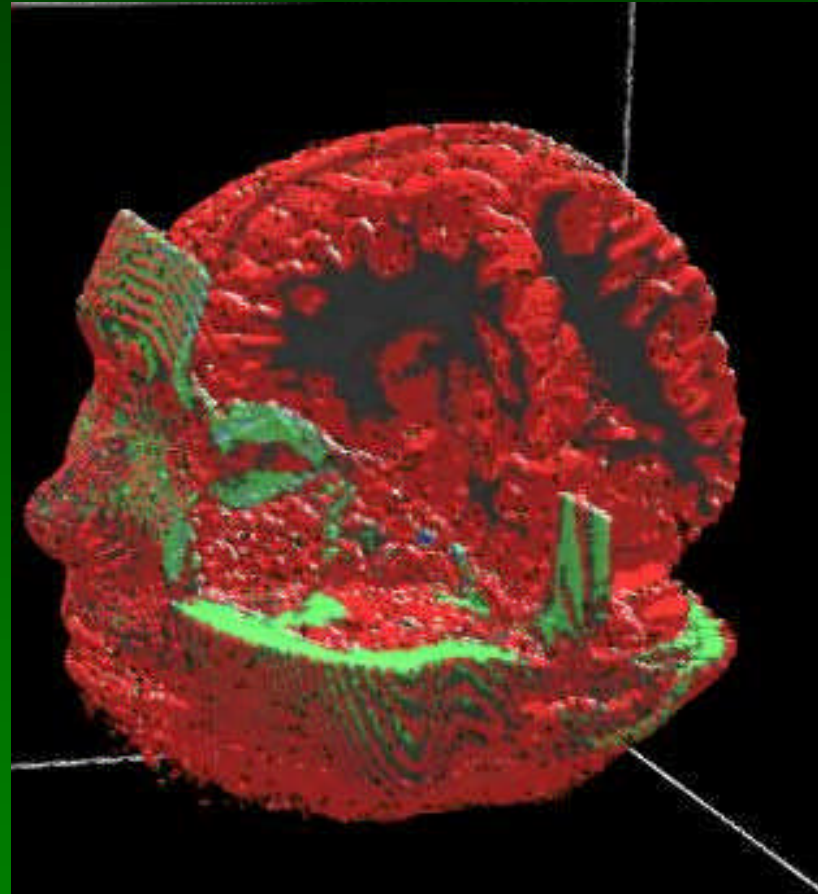
Example: 3D Textures

Emil Praun'01



Other Techniques

- Use CSG for cut-away



Acceleration of Volume Rendering

- Basic problem: Huge data sets
- Program for locality (cache)
- Divide into multiple blocks if necessary
 - Example: marching cubes
- Use error measures to stop iteration
- Exploit parallelism

Outline

- Height Fields and Contours
- Scalar Fields
- Volume Rendering
- **Vector Fields**

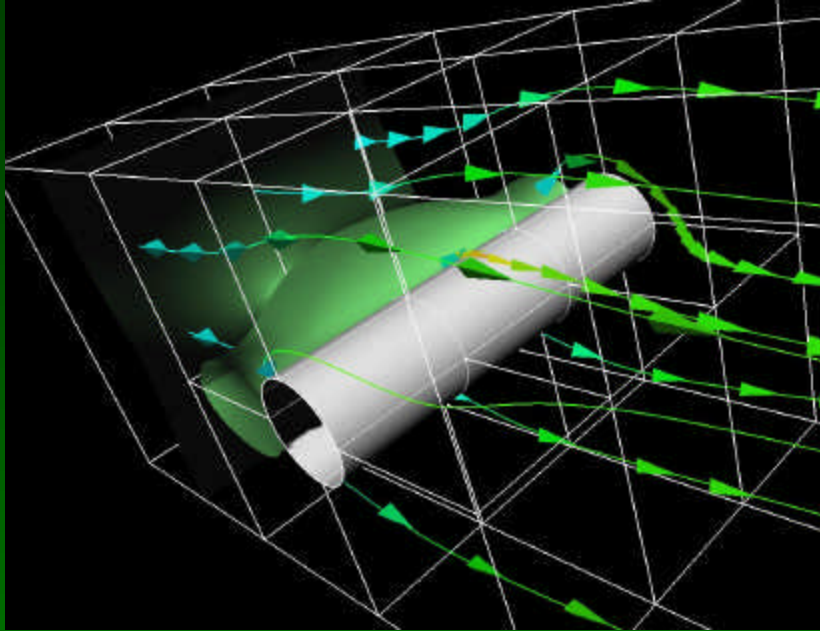
Vector Fields

- Visualize vector at each (x,y,z) point
 - Example: velocity field
 - Example: hair
- **Hedgehogs**
 - Use 3D directed line segments (sample field)
 - Orientation and magnitude determined by vector
- Animation
 - Use for still image
 - Particle systems

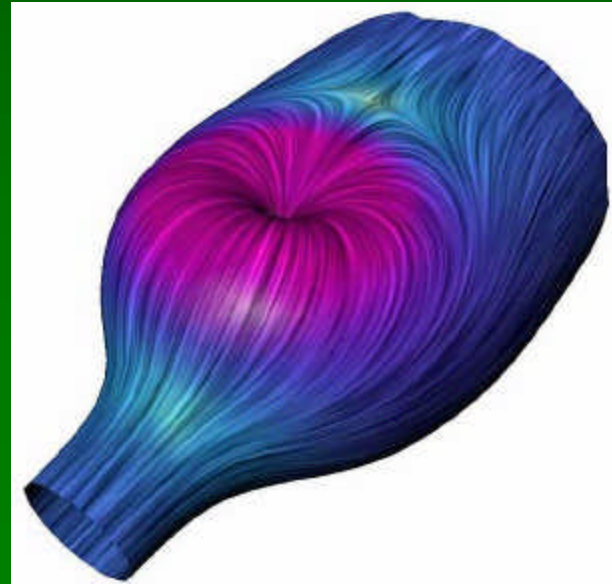
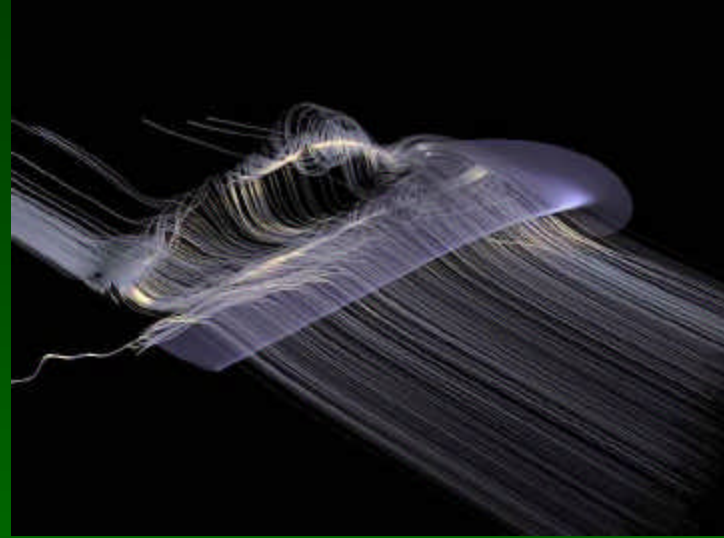


Blood flow in
human carotid artery

Using Glyphs and Streaks

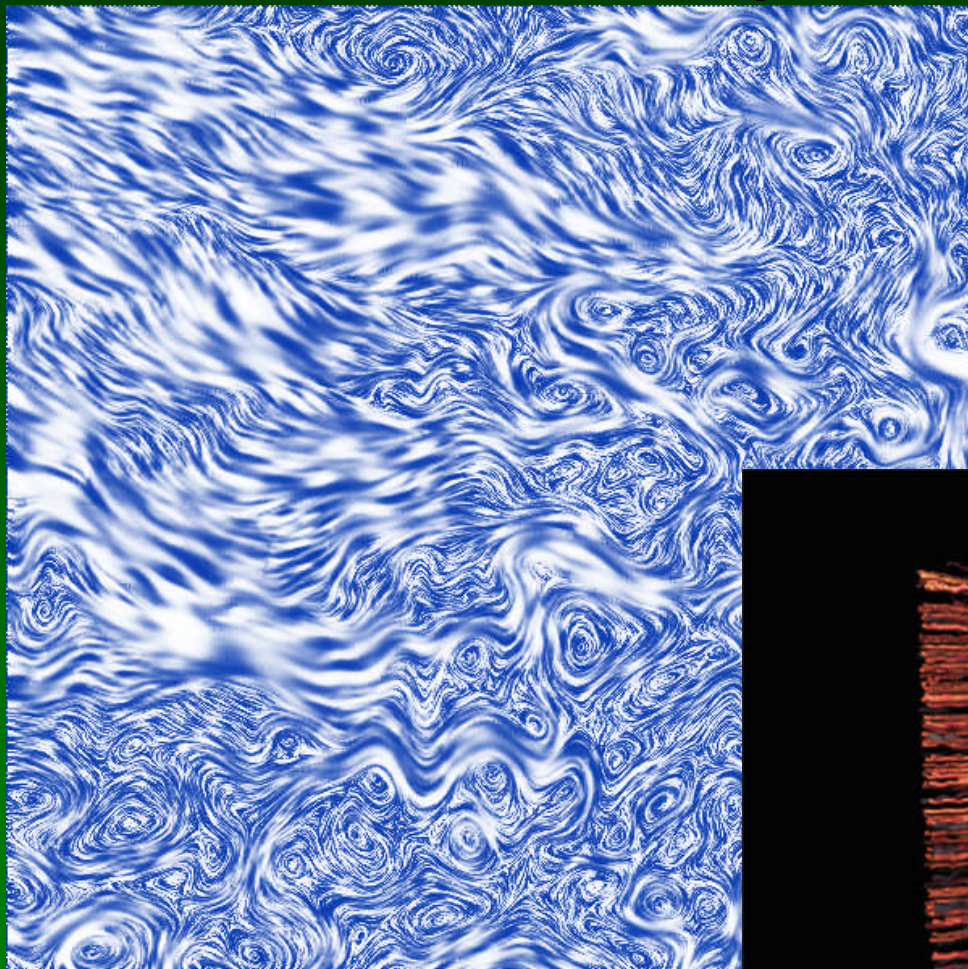


Glyphs for air flow

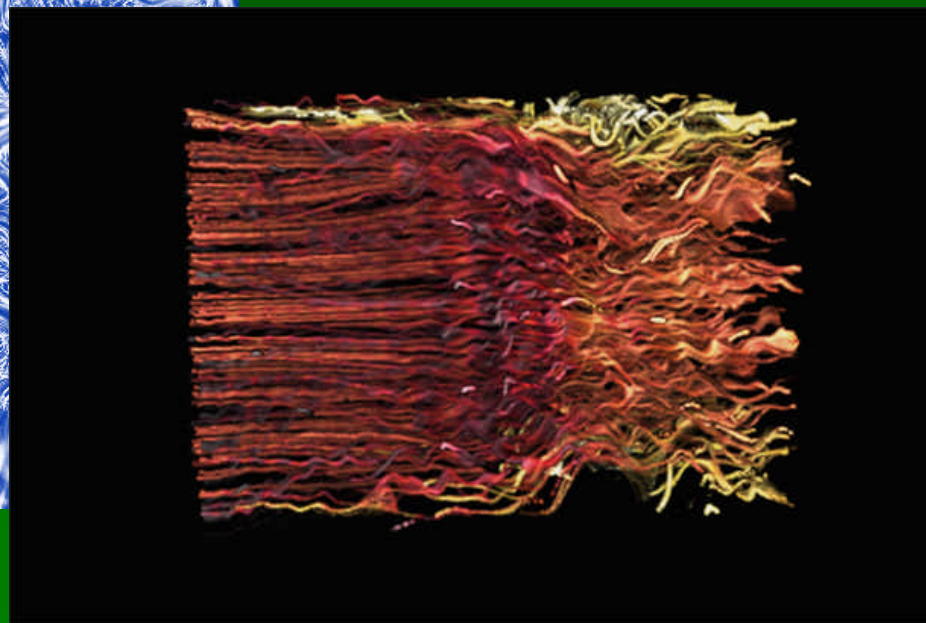


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More Flow Examples

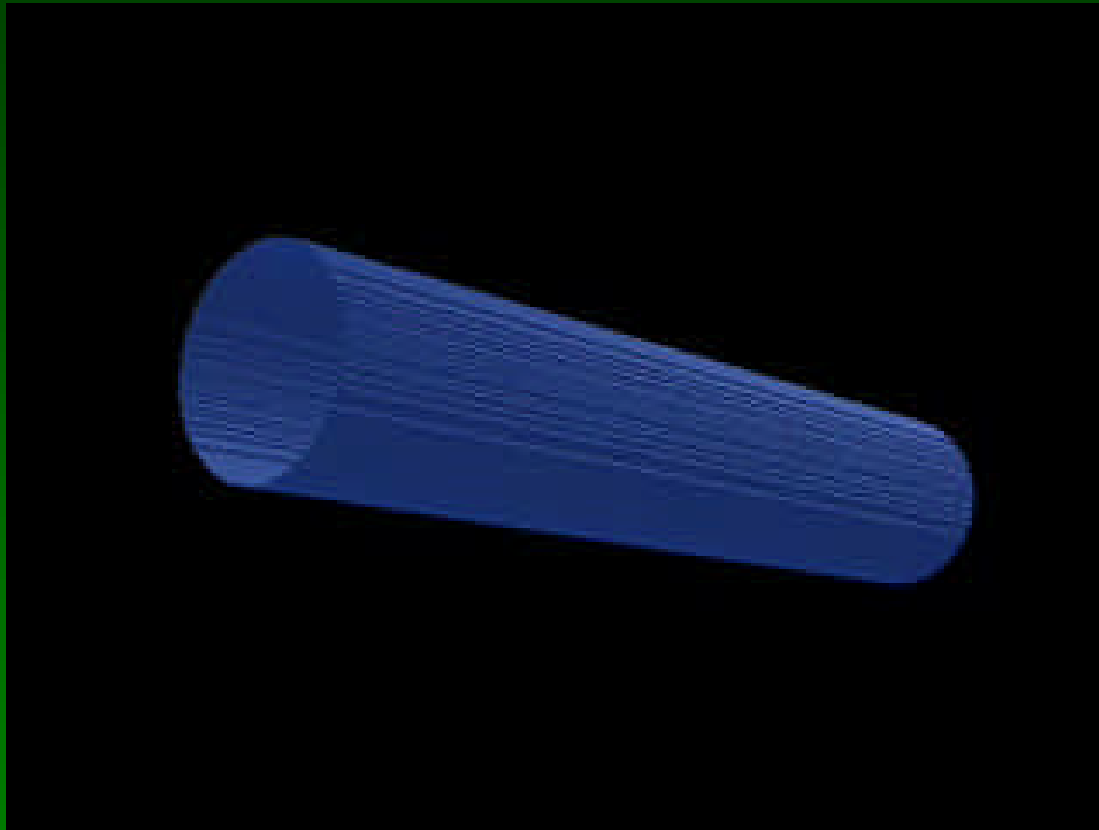


Banks and Interrante



Example: Jet Shockwave

P. Sutton
University of Utah



<http://www.sci.utah.edu/>

Summary

- Height Fields and Contours
- Scalar Fields
 - Isosurfaces
 - Marching cubes
- Volume Rendering
 - Volume ray tracing
 - Splatting
 - 3D Textures
- Vector Fields
 - Hedgehogs
 - Animated and interactive visualization

Preview

- Pick up Assignment 6
- Thursday
 - Non-photo-realistic rendering (NPR)
 - Assignment 7 (Ray Tracing) due!
 - Assignment 8 (written) out
- Next Tuesday
 - Guest lecture: Wayne Wooten, Pixar
- Next Thursday
 - Review for final
 - Assignment 8 due