15-213
"The Class That Gives CMU Its Zip!"

Bits and Bytes
January 13, 2005

Topics

- Why bits?
- Representing information as bits
  - Binary / Hexadecimal
  - Byte representations
    - Numbers
    - Characters and strings
    - Instructions
- Bit-level manipulations
  - Boolean algebra
  - Expressing in C

Why Don't Computers Use Base 10?

Base 10 Number Representation

- That's why fingers are known as "digits"
- Natural representation for financial transactions
  - Floating point number cannot exactly represent $1.20
- Even carries through in scientific notation
  - 15.213×10^3 (1.5213e4)

Implementing Electronically

- Hard to store
  - ENIAC (First electronic computer) used 10 vacuum tubes / digit
  - IBM 650 used 5+2 bits (1958, successor to IBM's Personal Automatic Computer, PAC from 1956)
- Hard to transmit
  - Need high precision to encode 10 signal levels on single wire
- Messy to implement digital logic functions
  - Addition, multiplication, etc.

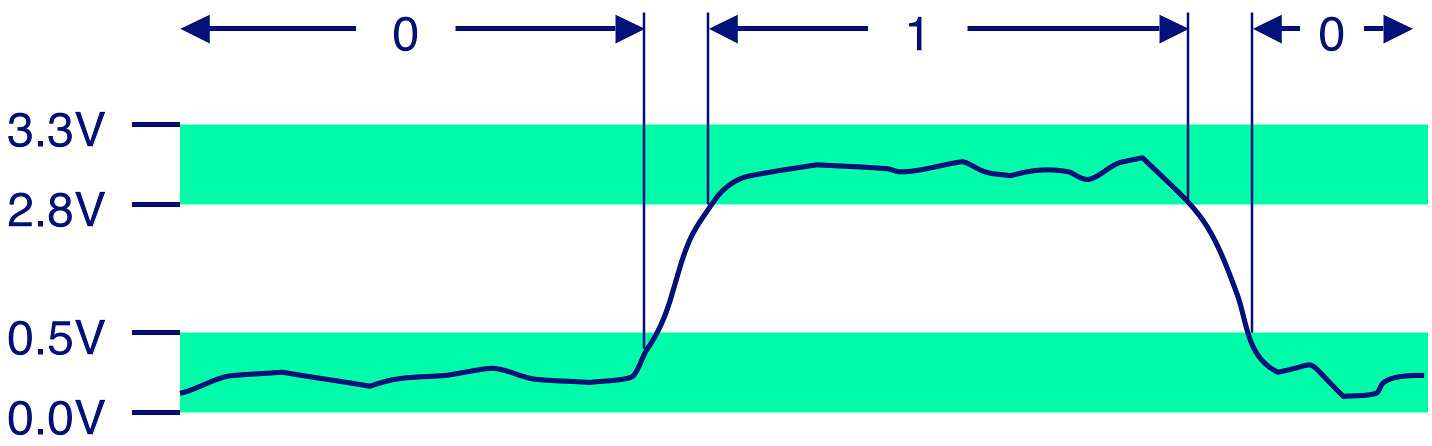
Binary Representations

Base 2 Number Representation

- Represent 15213_{10} as 11101101101101_2
- Represent 1.20_{10} as $1.0011001100110011[0011]..._2$
- Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

Electronic Implementation

- Easy to store with bistable elements
- Reliably transmitted on noisy and inaccurate wires



Byte-Oriented Memory Organization

Programs Refer to Virtual Addresses

- Conceptually very large array of bytes
- Actually implemented with hierarchy of different memory types
  - SRAM, DRAM, disk
  - Only allocate for regions actually used by program
- In Unix and Windows NT, address space private to particular "process"
  - Program being executed
  - Program can clobber its own data, but not that of others

Compiler + Run-Time System Control Allocation

- Where different program objects should be stored
- Multiple mechanisms: static, stack, and heap
- In any case, all allocation within single virtual address space

Encoding Byte Values

Byte = 8 bits

- Binary 00000000_2 to 11111111_2
- Decimal: 0_{10} to 255_{10}
  - First digit must not be 0 in C
- Octal: 000_8 to 0377_8
  - Use leading 0 in C
- Hexadecimal 00_{16} to FF_{16}
  - Base 16 number representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Write $FA1D37B_{16}$ in C as `0xFA1D37B`
    - » Or `0xfa1d37b`

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Literary Hex

Common 8-byte hex filler:

- `0xdeadbeef`
- Can you think of other 8-byte fillers?

Machine Words

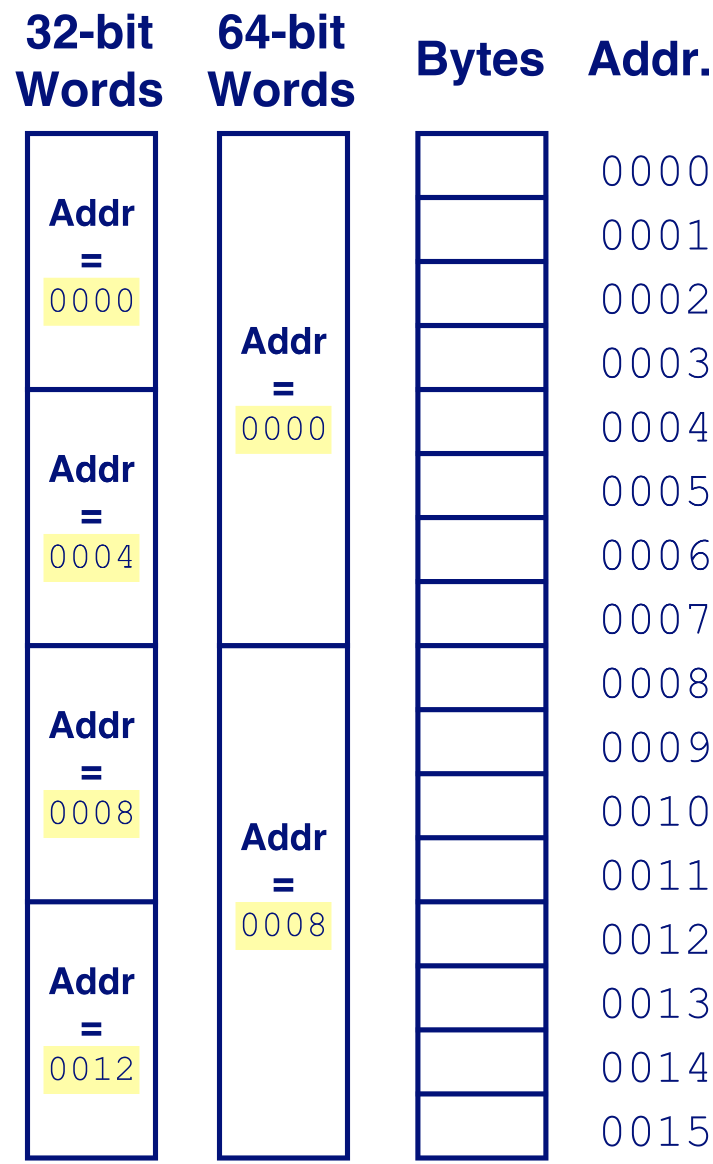
Machine Has "Word Size"

- Nominal size of integer-valued data
  - Including addresses
- Most current machines use 32 bits (4 bytes) words
  - Limits addresses to 4GB
  - Becoming too small for memory-intensive applications
- High-end systems use 64 bits (8 bytes) words
  - Potential address space ≈ 1.8×10^{19} bytes
- Machines support multiple data formats
  - Fractions or multiples of word size
  - Always integral number of bytes

Word-Oriented Memory Organization

Addresses Specify Byte Locations

- Address of first byte in word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Data Representations

Sizes of C Objects (in Bytes)

C Data Type	Alpha (RIP)	Typical 32-bit	Intel IA32
unsigned	4	4	4
int	4	4	4
long int	8	4	4
char	1	1	1
short	2	2	2
float	4	4	4
double	8	8	8
long double	8/16†	8	10/12
char *	8	4	4

» Or any other pointer

(†: Depends on compiler&OS, 128bit FP is done in software)

Byte Ordering

How should bytes within multi-byte word be ordered in memory?

Conventions

- **Suns, Macs (PPC) are "Big Endian" machines**
  - **Least significant byte has highest address**
- **Alphas, PC's are "Little Endian" machines**
  - **Least significant byte has lowest address**

Byte Ordering Example

Big Endian

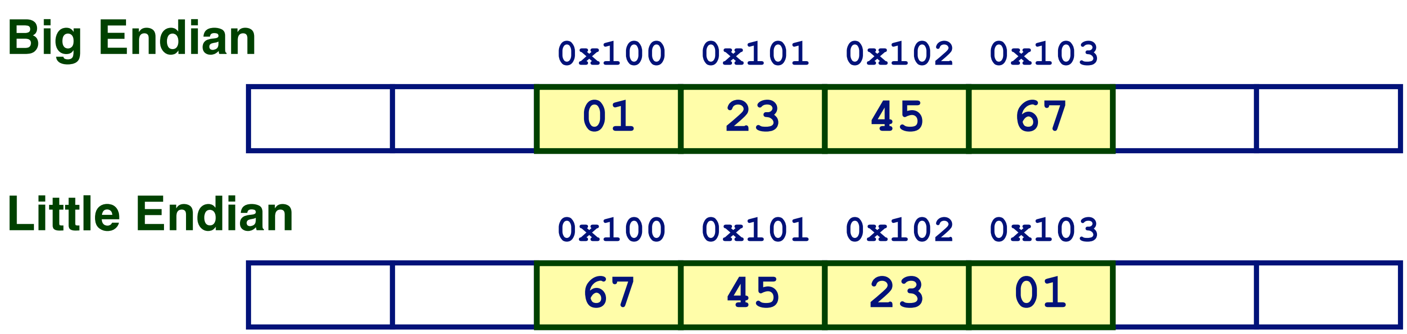
- Least significant byte has highest address

Little Endian

- Least significant byte has lowest address

Example

- Variable `x` has 4-byte representation `0x01234567`
- Address given by `&x` is `0x100`



Reading Byte-Reversed Listings

Disassembly

- Text representation of binary machine code
- Generated by program that reads the machine code

Example Fragment

Address	Instruction Code	Assembly Rendition	
`8048365:`	`5b`	`pop`	`%ebx`
`8048366:`	`81 c3 ab 12 00 00`	`add`	`$0x12ab,%ebx`
`804836c:`	`83 bb 28 00 00 00 00`	`cmpl`	`$0x0,0x28(%ebx)`

Deciphering Numbers

- Value: `0x12ab`
- Pad to 4 bytes: `0x000012ab`
- Split into bytes: `00 00 12 ab`
- Reverse: `ab 12 00 00`

Examining Data Representations

Code to Print Byte Representation of Data

- Casting pointer to `unsigned char *` creates byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len)
{
  int i;
  for (i = 0; i < len; i++)
    printf("0x%p\t0x%.2x\n",
           start+i, start[i]);
  printf("\n");
}
```

Printf directives:

`%p`: Print pointer

`%x`: Print Hexadecimal

show_bytes Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

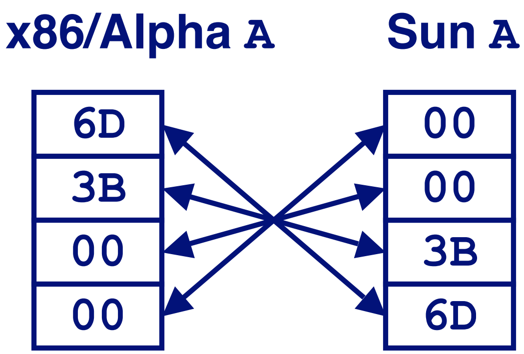
Result (x86 Linux):

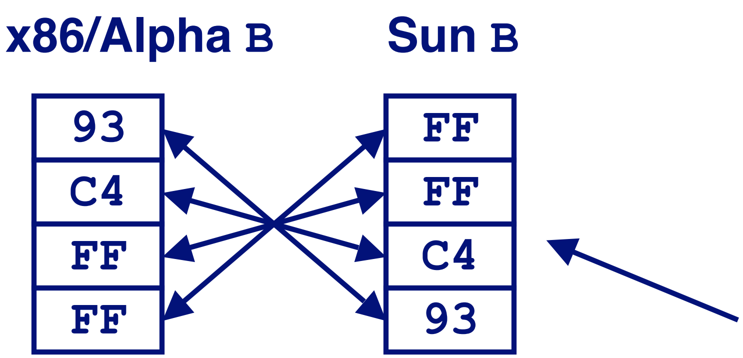
```
int a = 15213;
0x11ffffcb8   0x6d
0x11ffffcb9   0x3b
0x11ffffcba   0x00
0x11ffffcbb   0x00
```

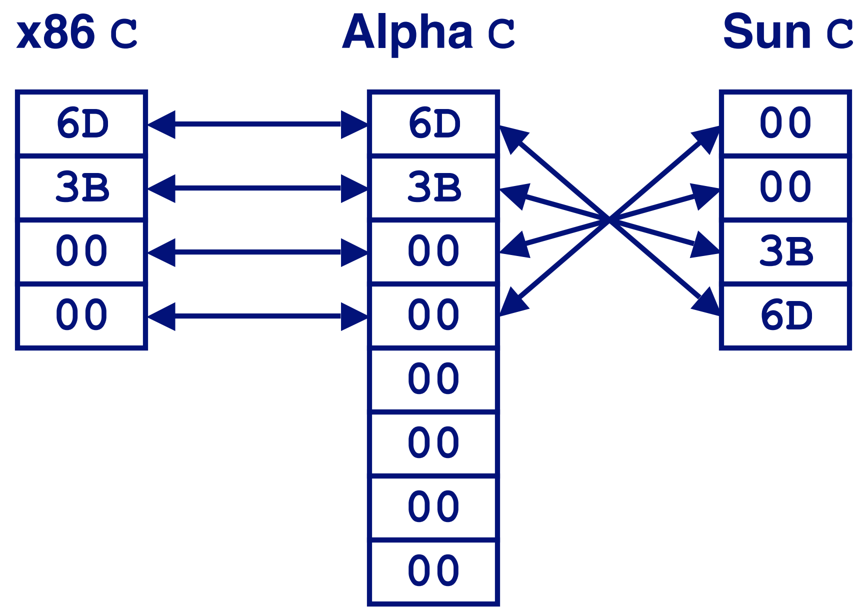
Representing Integers

```
int A = 15213;
int B = -15213;
long int C = 15213;
```

Decimal:	15213			
Binary:	0011	1011	0110	1101
Hex:	3	B	6	D







Two's complement representation (Covered next lecture)

Representing Pointers

```
int B = -15213;
int *P = &B;
```

Alpha P

A0
FC
FF
FF
01
00
00
00

Alpha Address

Hex:	1	F	F	F	F	F	C	A	0
Binary:	0001	1111	1111	1111	1111	1111	1100	1010	0000

Sun P

EF
FF
FB
2C

Sun Address

Hex:	E	F	F	F	F	B	2	C
Binary:	1110	1111	1111	1111	1111	1011	0010	1100

x86 Linux Address

Hex:	B	F	F	F	F	8	D	4
Binary:	1011	1111	1111	1111	1111	1000	1101	0100

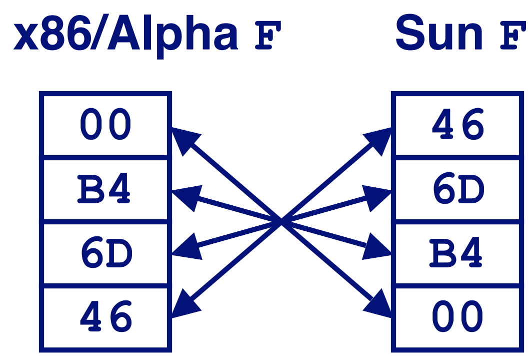
Linux P

D4
F8
FF
BF

Different compilers & machines assign different locations to objects

Representing Floats

`Float F = 15213.0;`



IEEE Single Precision Floating Point Representation

Hex:	4	6	6	D	B	4	0	0
Binary:	0100	0110	0110	1101	1011	0100	0000	0000
15213:			1110	1101	1011	01		

Not same as integer representation, but consistent across machines

Can see some relation to integer representation, but not obvious

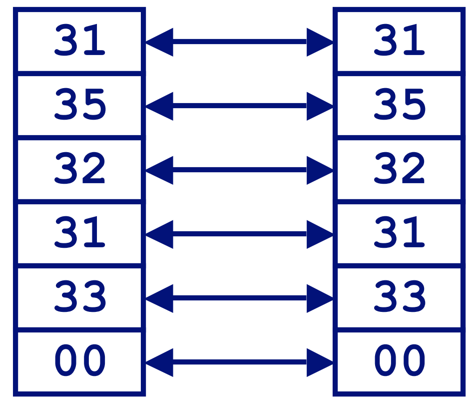
Representing Strings

```
char S[6] = "15213";
```

Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
  - Standard 7-bit encoding of character set
  - Character "0" has code `0x30`
    - » Digit *i* has code `0x30`+*i*
- String should be null-terminated
  - Final character = 0

x86/Alpha `S`   Sun `S`



Compatibility

- Byte ordering not an issue
- Text files generally platform independent
  - Except for different conventions of line termination character(s)!
    - » Unix and modern MacOS (`'\n' = 0x0a = ^J`)
    - » Older MacOS (`'\r' = 0x0d = ^M`)
    - » DOS and HTTP (`'\r\n' = 0x0d0a = ^M^J`)

Machine-Level Code Representation

Encode Program as Sequence of Instructions

- Each simple operation
  - Arithmetic operation
  - Read or write memory
  - Conditional branch
- Instructions encoded as bytes
  - Alphas, Suns, PPC use 4 byte instructions
    - » Reduced Instruction Set Computer (RISC)
  - PC's use variable length instructions
    - » Complex Instruction Set Computer (CISC)
- Different instruction types and encodings for different machines
  - Most code is not binary compatible

Programs are Byte Sequences Too!

Representing Instructions

```
int sum(int x, int y)
{
   return x+y;
}
```

- For this example, Alpha & Sun use two 4-byte instructions
  - Use differing numbers of instructions in other cases
- x86 uses 7 instructions with lengths 1, 2, and 3 bytes
  - Same for NT and for Linux
  - NT / Linux not fully binary compatible

Alpha `sum`	Sun `sum`	x86 `sum`
00	81	55
00	C3	89
30	E0	E5
42	08	8B
01	90	45
80	02	0C
FA	00	03
6B	09	45
		08
		89
		EC
		5D
		C3

Different machines use totally different instructions and encodings

Boolean Algebra

Developed by George Boole in 19th Century

- Algebraic representation of logic
  - Encode “True” as 1 and “False” as 0

And

- A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

- A|B = 1 when either A=1 or B=1

|	0	1
0	0	1
1	1	1

Not

- ~A = 1 when A=0

~	
0	1
1	0

Exclusive-Or (Xor)

- A^B = 1 when either A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

Integer Algebra

Integer Arithmetic

- $\langle Z, +, *, -, 0, 1\rangle$ **forms a "ring"**
- **Addition is "sum" operation**
- **Multiplication is "product" operation**
- **– is additive inverse**
- **0 is identity for sum**
- **1 is identity for product**

Boolean Algebra

Boolean Algebra

- ⟨{0,1}, |, &, ~, 0, 1⟩ **forms a "Boolean algebra"**
- **Or is "sum" operation**
- **And is "product" operation**
- **~ is "complement" operation (not additive inverse)**
- **0 is identity for sum**
- **1 is identity for product**

Boolean Algebra ≈ Integer Ring

- *Commutativity*

Boolean Algebra	Integer Ring
A | B = B | A	A + B = B + A
A & B = B & A	A * B = B * A

- *Associativity*

Boolean Algebra	Integer Ring
(A | B) | C = A | (B | C)	(A + B) + C = A + (B + C)
(A & B) & C = A & (B & C)	(A * B) * C = A * (B * C)

- *Product distributes over sum*

Boolean Algebra	Integer Ring
A & (B | C) = (A & B) | (A & C)	A * (B + C) = A * B + B * C

- *Sum and product identities*

Boolean Algebra	Integer Ring
A | 0 = A	A + 0 = A
A & 1 = A	A * 1 = A

- *Zero is product annihilator*

Boolean Algebra	Integer Ring
A & 0 = 0	A * 0 = 0

- *Cancellation of negation*

Boolean Algebra	Integer Ring
~ (~ A) = A	– (– A) = A

Boolean Algebra ≠ Integer Ring

- **Boolean:** ***Sum distributes over product***

  A | (B & C) = (A | B) & (A | C)   A + (B * C) ≠ (A + B) * (A + C)

- **Boolean:** ***Idempotency***

  A | A = A   A + A ≠ A

  - "A is true" or "A is true" = "A is true"

  A & A = A   A * A ≠ A

- **Boolean:** ***Absorption***

  A | (A & B) = A   A + (A * B) ≠ A

  - "A is true" or "A is true and B is true" = "A is true"

  A & (A | B) = A   A * (A + B) ≠ A

- **Boolean:** ***Laws of Complements***

  A | ~A = 1   A + –A ≠ 1

  - "A is true" or "A is false"

- **Ring:** ***Every element has additive inverse***

  A | ~A ≠ 0   A + –A = 0

Properties of & and ^

Boolean Ring

- ⟨{0,1}, ^, &, I, 0, 1⟩
- Identical to integers mod 2
- I is identity operation: I(A) = A

  A ^ A = 0

Property	Boolean Ring
Commutative sum	A ^ B = B ^ A
Commutative product	A & B = B & A
Associative sum	(A ^ B) ^ C = A ^ (B ^ C)
Associative product	(A & B) & C = A & (B & C)
Prod. over sum	A & (B ^ C) = (A & B) ^ (A & C)
0 is sum identity	A ^ 0 = A
1 is prod. identity	A & 1 = A
0 is product annihilator	A & 0 = 0
Additive inverse	A ^ A = 0

Relations Between Operations

DeMorgan's Laws

- Express & in terms of |, and vice-versa
  - A & B = ~(~A | ~B)
    - » A and B are true if and only if neither A nor B is false
  - A | B = ~(~A & ~B)
    - » A or B are true if and only if A and B are not both false

Exclusive-Or using Inclusive Or

- A ^ B = (~A & B) | (A & ~B)
  - » Exactly one of A and B is true
- A ^ B = (A | B) & ~(A & B)
  - » Either A is true, or B is true, but not both

General Boolean Algebras

Operate on Bit Vectors

- Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

All of the Properties of Boolean Algebra Apply

Representing & Manipulating Sets

Representation

- **Width** w **bit vector represents subsets of** $\{0, \ldots, w-1\}$
- $a_j = 1$ **if** $j \in A$

```
01101001        { 0, 3, 5, 6 }
76543210

01010101        { 0, 2, 4, 6 }
76543210
```

Operations

- **& Intersection** 01000001 { 0, 6 }
- **| Union** 01111101 { 0, 2, 3, 4, 5, 6 }
- **^ Symmetric difference** 00111100 { 2, 3, 4, 5 }
- **~ Complement** 10101010 { 1, 3, 5, 7 }

Bit-Level Operations in C

Operations &, |, ~, ^ Available in C

- Apply to any "integral" data type
  - `long, int, short, char, unsigned`
- View arguments as bit vectors
- Arguments applied bit-wise

Examples (Char data type)

- `~0x41 --> 0xBE`

  $\sim 01000001_2 \rightarrow 10111110_2$
- `~0x00 --> 0xFF`

  $\sim 00000000_2 \rightarrow 11111111_2$
- `0x69 & 0x55 --> 0x41`

  $01101001_2 \,\&\, 01010101_2 \rightarrow 01000001_2$
- `0x69 | 0x55 --> 0x7D`

  $01101001_2 \mid 01010101_2 \rightarrow 01111101_2$

Contrast: Logic Operations in C

Contrast to Logical Operators

- `&&, ||, !`
  - View 0 as "False"
  - Anything nonzero as "True"
  - Always return 0 or 1
  - Early termination

Examples (char data type)

- `!0x41  -->  0x00`
- `!0x00  -->  0x01`
- `!!0x41 -->  0x01`

- `0x69 && 0x55  -->  0x01`
- `0x69 || 0x55  -->  0x01`
- `p && *p` (avoids null pointer access)

Shift Operations

Left Shift: `x << y`

- Shift bit-vector `x` left `y` positions
  - Throw away extra bits on left
  - Fill with 0's on right

Right Shift: `x >> y`

- Shift bit-vector `x` right `y` positions
  - Throw away extra bits on right
- Logical shift
  - Fill with 0's on left
- Arithmetic shift
  - Replicate most significant bit on right
  - Useful with two's complement integer representation

Argument `x`	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument `x`	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

Cool Stuff with Xor

- Bitwise Xor is form of addition
- With extra property that every value is its own additive inverse

  A ^ A = 0

```
void funny(int *x, int *y)
{
    *x = *x ^ *y;    /* #1 */
    *y = *x ^ *y;    /* #2 */
    *x = *x ^ *y;    /* #3 */
}
```

	*x	*y
Begin	A	B
1	A^B	B
2	A^B	(A^B)^B = A
3	(A^B)^A = B	A
End	B	A

More Bitvector Magic

Count the number of 1's in a word

MIT Hackmem 169:

```
int bitcount(unsigned int n)
{
    unsigned int tmp;

    tmp = n - ((n >> 1) & 033333333333)
            - ((n >> 2) & 011111111111);
    return ((tmp + (tmp >> 3)) & 030707070707)%63;
}
```

Some Other Uses for Bitvectors

Representation of small sets

Representation of polynomials:

- Important for error correcting codes
- Arithmetic over finite fields, say GF(2^n)
- Example 0x15213 : $x^{16} + x^{14} + x^{12} + x^9 + x^4 + x + 1$

Representation of graphs

- A ‘1’ represents the presence of an edge

Representation of bitmap images, icons, cursors, …

- Exclusive-or cursor patent

Representation of Boolean expressions and logic circuits

Summary of the Main Points

It's All About Bits & Bytes

- Numbers
- Programs
- Text

Different Machines Follow Different Conventions for

- Word size
- Byte ordering
- Representations

Boolean Algebra is the Mathematical Basis

- Basic form encodes "false" as 0, "true" as 1
- General form like bit-level operations in C
  - Good for representing & manipulating sets